

Analysis Of Students' Difficulties In Mathematical Proof Ability Viewed From An Epistemological Aspect

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Abstrak. The ability to prove is the essence in studying mathematics. And the ability to prove mathematics has not fully grown in students. This study specifically aims to analyze the ability of mathematical proof, to analyze learning difficulties in terms of student epistemology on the Limit Function material. The long-term benefit of this research is that the study of learning difficulties in terms of student epistemology related to mathematical proof in the Real Analysis course is expected to provide encouragement to other lecturers to further develop the learning process or teaching materials in an effort to develop mathematical proof skills for education students. mathematics. This study used a descriptive method, while the research subjects were 9 prospective mathematics teacher students at Pancasakti Tegal University who contracted the Real Analysis course. Data collection methods used include: (1) test of mathematical proof ability; (2) observation; (3) interview; and (4) documentation. The results obtained that there are 4 kinds of student difficulties in terms of epistemology related to Real Analysis courses, namely: a) learning difficulties related to difficulties in applying concepts; b) learning difficulties related to difficulties in determining principles; c) learning difficulties related to understanding the problem and d) related to difficulties in mathematical proof. Especially in mathematical proof, students experience difficulties, among others: not knowing how to start constructing proofs, not being able to use definitions (concepts) and principles that are already known, and tend to start constructing proofs with what has to be proven.

Keywords: Learning Difficulty, Evidence Ability, Epistemology

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INTRODUCTION

One of the courses that suit some of the goals of college mathematics is the Real Analysis course. Syawahid (2015) said that Real Analysis is the first course for students to practice reasoning and formally prove mathematical statements. Although real analysis is a very important course, most mathematics students still consider it a difficult subject (Astawa et al, 2018; isnani, 2019a; Moore, 1994; Tall, 1998; Sugilar, 2015). Septian (2014) also explained that a lot of material in real analysis is assessed as abstract material so that many students have difficulty understanding Real Analysis courses (Astawa, 2016; Isnani, 2019; Isnani, 2020b; Isnani, 2020c; Isnani, 2021). said that the difficulties experienced by students in the learning process include not being aware of the consequences of a theorem, difficulty in providing counter examples, lack of understanding of implications, lack of understanding of proof using definitions, and difficulties in performing algebraic forms. Meanwhile (Kartini, 2015; Selden & Selden, 2003; Sundawa, 2018) said that students' difficulties in constructing evidence include lack

of understanding of sets and logic, lack of knowledge of various proofing techniques, unable to use premises, unable to use existing definitions or theorems, difficulty understanding what will be proven, difficulty writing sentences in language and mathematical notation, difficulty in algebraic manipulation, and lack of understanding of prerequisite material.

Hana (Christou, 2004), says that the functions of evidence and proof are: verification, explanation, systematization, discovery, communication, construction, exploration and amalgamation. (verification, explanation, systematization, discovery, communication, construction, exploration, and incorporation). Verification of proof and proof is considered the most fundamental function in proof because they are both products of a very mature process of developing mathematical thinking. Verification refers to the truth of a statement while explanation provides insight into why the statement is true.

One of the materials in the Real Analysis course is the limit function, (Bartle, 2001; Bartle, 2011; Dwijanto, 1994). Students' understanding of the concept of limits shows that

students have a smooth conceptual representation of limits (Davis & Vinner, 1986; Karatas et al., 2011; Beynon & Zollman, 2015). Most students have trouble applying the concept of limits intuitively into formal concepts. Many students do not understand the formal definition of limit as a statement that is equivalent to what they have intuitively learned (Row, 2007; Kim et al., 2015). In this material, students experience difficulties in proving skills including, lack of learning experience, and lack of strategies in proving.

Based on these problems, learning difficulties will be studied in terms of student epistemology related to the ability to prove mathematically in the Real Analysis course for the Limit Function material. This study is to find an explanation of the students' mathematical proof ability in the Real Analysis course for the Limit Function material and to find out students' learning difficulties in terms of student epistemology related to the Limit Function material.

METHODS

This research is a type of qualitative research with a descriptive approach. The aim of the research is to describe learning difficulties in terms of student epistemology related to mathematical proof in the Real Analysis course for Limits of Functions material. The research subjects were 9 prospective mathematics teacher students at Pancasakti University of Tegal who contracted the Real Analysis course consisting of 3 students with high initial mathematical abilities, 3 students with moderate initial mathematical abilities and 3 students with low initial mathematical abilities. Subject taking technique using purposive sampling technique. This initial knowledge is based on achievement in Basic Calculus courses. For student learning difficulties in terms of student epistemology in transformation material using 5 indicators namely concepts, visualization, principles, understanding problems, and mathematical proof. The methods for collecting data in this study are the documentation method, the test method, and the interview method. The test method is used in the form of a mathematical proving ability test. The main instruments in this study were the researchers themselves and interview guides. Data analysis in this study included data reduction, data presentation, and drawing conclusions, while testing the validity of the data in this study used technical and time triangulation techniques.

RESULTS AND DISCUSSION

Based on the results of the study, it was obtained that the proving ability of students of Mathematics Education at Pancasakti University, Tegal in the Real nasis course for the Limit Function material varied. There are those who have high, medium, and low abilities in this proof.

The difficulties of students in understanding the concept of the limit function concept from an epistemological perspective in working on Real Analysis questions on the Limit Function material are:

Difficulty understanding and applying concepts

Students do not understand the concept of limits. Students memorize the definition of the limit of a function and the definition of the concept of limit, but do not know the use of the definition of the limit of the function. This happens to students who have moderate and low initial mathematical abilities. The difficulty of students with high, medium and low abilities is the difficulty in taking the delta that will be used. However, those with high abilities can finish it to completion. While those who have medium and low abilities in solving incomplete problems. Some students do not understand the concept of limits correctly, so when working on questions they are still confused. This is like the results of research from (Arora, 2021; Isnani, 2020a; Isnani, 2020d; Kim et al., 2015; Oktaviyanthi, 2018; Row, 2007;) which says that most students have problems applying the concept of limits intuitively into formal concepts. Many students do not understand the formal definition of limit as a statement equivalent to what they have intuitively learned. Following are the results of interviews with students, namely:

T : Why are you experiencing difficulties in understanding the concept

M₂ : I only memorize the concept of limit but don't know how to use it

S₃ : Same with me too

L₂ : I understand the concept and its usage

T : Why is everything in use there is still something wrong

L₁ : Yes, ma'am, I was immediately proven, so I took the wrong delta, I should have used the initial analysis to make things easier

M₃ : Penggunaanya lupa bu

S₁ : I don't study ma'am

Difficulty visualizing objects

Difficulty visualizing function limit objects. It means that students have difficulty in describing in Cartesian coordinates, to determine the position of the domain area and the result area. High ability students can visualize according to the questions asked correctly, while students with moderate abilities have difficulty visualizing. Then for low abilities do not do drawing in this visualization. Kesulitan memvisualisasi dikarenakan mahasiswa kurang representasi yang kurang formal. proof writing (Raman, 2003) atau mereka tidak mampu menerjemahkan wawasan visual mereka ke dalam bahasa pembuktian (Zazkis, 2014). Similarly, considering specific examples of general claims can provide insights that can be useful for writing evidence (e.g., Lockwood et al., 2012; Sandefur et al., 2013). The results of interviews with students are as follows:

T : What are the obstacles in the visualization

S₂ : I don't understand at all ma'am

M₂ : I'm still unsure about visualizing

L₁ : When studying, I already understand well so I can do it properly

Difficulty determining principles

This difficulty is a difficulty experienced by students in terms of solving problems by determining the principles that will be used in solving function limit problems and the concept of limit concepts. The delta used is still in the domain area. Students of medium and low ability have difficulty in determining the delta. Students lack experience in proving as well as lack of skills and strategies in proving. This is also as described by (Moore, 1994; Weber, 2010). Likewise, according to (Reif, 2008) with experience in proof one day can give birth to a strategy of proof and a strategy of proof. Likewise, according to (Reif, 2008) with experience in proving one day can give birth to a strategy of proof. According to students, the difficulties are:

T : Where lies the difficulty in determining the principle

L₃ : I often work on the questions in the book, over time I have my own way or I use it as my strategy

M₁ : I learn by rote so if I ask to do the questions I get confused

S₂ : Saya tidak tahu penggunaan dan maupun prinsipnya

T : Why don't you know

S₁ : It's a headache when learning Real Analysis is too Abstract

Difficulty understanding the problem

The difficulties experienced by students in terms of understanding the problem to solve the problem by using the completion steps that are in accordance with the definition of the limit of the function. Obtained for high ability to complete according to existing procedures correctly. Meanwhile, moderate ability can complete the procedure but it is not right. Low ability can not complete the procedure and is not precise. This difficulty is caused by students not understanding the definition and concept of limit. As experienced in research (Amaliyakh, 2015; Hart, 1994; Tall & Vinner, 1981) it was found that students had difficulties with students' understanding of concepts and definitions. This difficulty has demonstrated the intricate interaction between the stages of understanding, planning, and implementing plans. According to (Carlson & Bloom, 2005) it is indeed the plan above to understand some aspects of the problem that confuses them. Discussion with students the difficulties are:

T : Where is the difficulty in understanding the definition?

M₃ : I memorize and understand the definition and steps to prove it, but I'm confused about using the language

S₁ : I don't understand the definition well

T : Why

M₁ : We are still unsure about the language to be used for each step

S₃ : I can't understand the notation notation in the definition

Difficulty of mathematical proof

The difficulties experienced by students in constructing evidence from questions. Obtained for high ability students can construct evidence correctly, but there are errors in writing notation. Moderate ability can construct partial evidence, at the end of the section there is an error in giving reasons. Low abilities cannot use definitions to construct evidence. According to (Sumarmo, 2004) constructing evidence is the ability to compile a proof of a mathematical statement based on definitions, principles, and theorems, and write it down in the form of a complete proof. Students express their difficulties, namely:

T : Why are you experiencing difficulties in constructing evidence

L₃ : I forgot to write down the notation in the proof at the end

M₂ : I forgot not to do the initial analysis so as to claim the magnitude of the delta had an error as a result the final result was also wrong

S₃ : I don't understand at all in the proof

T : To make it easier to construct evidence, it would be better to do a preliminary analysis beforehand so that valid final results are obtained

CONCLUSION

The conclusion in this study is that students' proving abilities vary. There are 5 kinds of student difficulties in terms of epistemology related to Real Analysis course for Limit Function material, namely a). difficulty in applying concepts; b). Difficulty visualizing objects; c). Difficulty determining principles; d). Difficulty understanding the problem; e). Difficulty of mathematical proof.

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