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Analysis of Student's Misconceptions in Solving a Discrete Random Variable

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Abstract. One of the most important obstacles in learning mathematics is misconceptions. This study aims to analyze student misconceptions in solving the problem of one discrete random variable in probability theory courses. The research was conducted on students of semester fifth of mathematics education study program at Indraprasta University PGRI Jakarta who took probability theory courses. The method in this study is qualitative. The study subjects were two people selected using snowball sampling techniques. The instruments used are tests on probability theory courses, interview guidelines, and observations. Testing the validity of research data using triangulation. Data analysis is done using data presentation, data reduction, and conclusion withdrawal. The results showed that the misconceptions experienced by subjects in discrete random variable material, namely in the process of determining the function of probability. First subject (S1) assumes that the probability function is equal to the probability value, whereas Second subject (S2) cannot distinguish properties on geometric and binomial discrete special distributions. Such misconceptions lead to a constant misconception in determining the function of probabilities. This will result in other misconceptions related to the probability function material, such as determining expectations and variances.

Key words: misconceptions; one discrete random variable; probability theory.

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INTRODUCTION

The goal of education and mathematics learning at the college level is to develop high-level thinking skills. This is necessary as a provision for prospective students of mathematics teachers to be able to teach and encourage the mathematical thinking skills of students that they will teach. One course that requires good mathematical thinking skills is probability theory. Probability theory is a course that must be followed by students in mathematics education courses. Probability theory is a complicated course, which has systematic and logical characteristics (Wang &Xu, 2018). Probability can be described by a formula, a graph, in which each event is linked to its probability. This kind of description of probability is called probability distribution (Viti et al., 2015). Discrete probability distribution (applicable to the scenarios where the set of possible outcomes is discrete, such as a coin toss or a roll of dice) can be encoded by a discrete list of the probabilities of the outcomes, known as a probability mass function (Malakar, 2020).

However, the learning achievements in probability theory courses have not been as expected. One of the obstacles in learning probability theory is that students still have difficulty in solving questions that are abstract concepts. One indication of not achieving the optimal learning objectives of mathematics is the problem of student misconceptions. Regarding the difficulties associated with probability theory, previous research has stated that students have misunderstandings for various reasons (Koparan, 2015) one reason is that problems arise when the concept of the prerequisite material is not well mastered. Other research shows that the difficulty occurs because students do not know the concept, the interrelationship between concepts, and how to apply the concept in real problems (Yusuf et al., 2019).

In working on the problem related to probability theory, it is necessary to understand the concept of good prerequisite material. For example, to learn and work on problems on discrete random variable materials, students must master prerequisite materials such as rows and sequences that have been studied at the secondary school level. Then, to learn the continuous random variable material, students must understand the integral concepts that have been studied in calculus courses. However, the difficulties found are related to all its basic ideas: for example variables, functions, boundaries, derivatives, integrals, and differential equations (Lithner, 2011) are also experienced by students. As a result, concepts that have a true value can be considered a student simply because of different symbols or different order of writing symbols.

Therefore, lecturers need to understand student misconceptions (Ojose, 2015); (Jankvist &Niss, 2018); (Mulyatna et al., 2020).

Misunderstanding in mathematics is the process of forming a disturbing new concept. (Özkan, 2011). There has been a lot of research on misconceptions in mathematics learning (Ming et al., 2017). Mistakes can be miscalculations or misjudgments and such categories fall under unsystematic errors (Ming et al., 2017). For example, misconceptions are committed by students in calculus due to a lack of basic knowledge in algebra and mathematical thinking skills (Ming et al., 2017). Students often have difficulty implementing thinking and solving methods in learning the learning process, many students are unable to digest knowledge (Wang &Xu, 2018). Misconceptions include understanding or thinking that is not based on correct information (Kusmaryono et al., 2020). According to Dayanti, Sugiatno, &Nursangaji (2019), there are three types of misconceptions that students usually do; classification misunderstandings, correlational misunderstandings, and theoretical misunderstandings.

On that basis, the purpose of the study is to further analyze the misconceptions in students in solving the problem of one discrete random variable in the course of probability theory. Many papers are devoted to the concept of probability but not so much devoted to the concept of random variables (Kachapova &Kachapov, 2012). The findings of this study are expected to be a reference for practitioners of mathematics education to overcome student misconceptions in studying probability theory courses, especially those related to the question of one discrete random variable. Through this article, it is expected to recommend things that are considered to be able to prevent students from misconceptions.

METHODS

Research Goal

This study aims to analyze student misconceptions in solving the problem of one discrete random variable in probability theory courses.

Sample and Data Collection

The research was conducted at Indraprasta University PGRI Jakarta, the odd semester of the academic year 2020/2021. The method used in this study is qualitative, where the activity is carried out in the condition of the natural object. Sampling techniques use snowball sampling and the data source comes from students as research subjects. For the determination of samples, one person was first chosen, because one person has not provided enough data then another person was chosen (Sugiyono, 2016). Thus, the subject was taken as many as two students, selected from students of mathematics education semester 5 the academic year 2020/2021 who took the course of probability theory. Both subjects have been able to provide information about misconceptions made in solving a discrete random variable. The instruments used are tests on probability theory courses, interview guidelines, and observations. For test instruments, it has been validated in advance by experts so that it is ready to be tested on the research subject. Data collection techniques were performed through triangulation.

Analyzing of Data

Data analysis using the Miles and Huberman model, conducted through data reduction, data presentation, verification, and the final stage of concluding. Data analysis is used to describe and conclude the results in answering existing problems (Supandi et al., 2019).

RESULTS AND DISCUSSION

The material of one discrete random variable is studied in the probability theory courses. The learning objective of this material is that students are expected to be able to: 1) determine a function of probability for a discrete random variable based on its nature; 2) determine the distribution of probability from a discrete random variable and its modifications; 3) determine the odds of a particularly valuable discrete random variable; 4) determine the expectation and variance values of a discrete random variable.

Here's a question that was given to students about a discrete random variable of material about the function of probability.

Question. A coin is made in such a way that the appearance of image side events is 2 times more frequent than the number side events. Then the coin is chanted continuously until the appearance of the first image-side event on the 10th guide. What does the probability of the event do?

In the above question, the problem given regarding the probability function of a currency throw is unbalanced. To answer these questions, students can use one of the discrete special distributions that are geometric distribution. The geometric distribution is a distribution that has the nature of experimentation repeated several times until the first successful event occurs The definition of geometric probability function (Herrhyanto &Gantini, 2014) is.

Random Variable X has geometrically distributed words, if and only if the probability function is shaped: $p(x) = P(X = x) = (1 - p)^{x-1}.p; x = 1,2,3, \ldots \ldots$

So, to solve the problem, due to the appearance of the image side events (G), twice as often as a number side occurrence (A), then each of the probabilities is P(G) = $\frac{2}{3}$ and P (A) = $\frac{1}{3}$. Then, the appearance of image-side events is regretted with successful events and the appearance of the number side is a failed event. Thus, if using geometric distribution, the function of opportunities from the occurrence of continuous coin chanting until the appearance of the first image-side occurrence in the tenth chant is.

$p(x) = P(X = x) = (1 - p)^{x-1}.p; x = 1,2,3, \ldots \ldots ,10$

$$p(x) = \left(1 - \frac{2}{3}\right)^{x-1} . \frac{2}{3}; x = 1,2,3, \ldots \ldots, 10 \quad (1)$$

or

$$p(x) = \frac{2}{3} \quad ; x = 1$$

$$p(x) = \frac{1}{3}.\frac{2}{3} = \frac{2}{9} \quad ; x = 2$$

$$p(x) = \left(\frac{1}{3}\right)^2 . \frac{2}{3} = \frac{2}{27} \quad ; x = 3$$

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$$p(x) = \left(\frac{1}{3}\right)^9 . \frac{2}{3} = \frac{2}{59.049} \quad ; x = 10 \quad (2)$$

The general form of a probability function can be written into the form of a function of a random variable value (1) or a probability function that is a constant and consists of more than one value (2), meaning that for each random change value assigned each, it has its probability function value. The misconception for question number 1 is derived from the understanding that to determine the function of a probability is the same as determining the value of the probability so that most students directly look for the value of the probability without writing down the function of the probability. As can be seen in the following image.

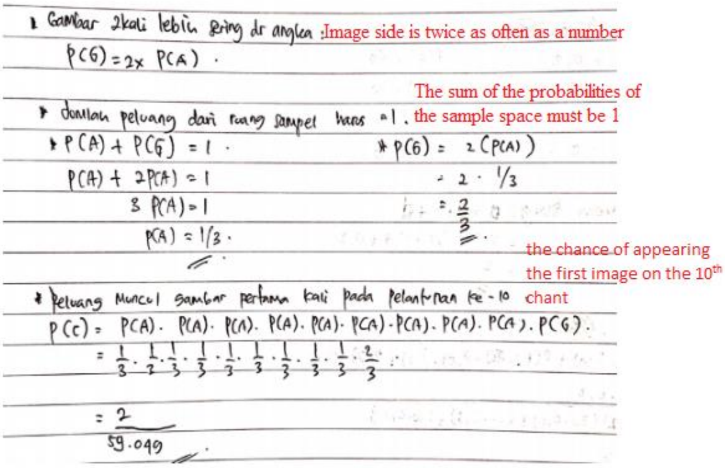
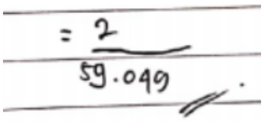


Figure 1. Answer S1

Based on figure 1, S1 can determine each probability value of the occurrence of numbers and the appearance of images, then can describe the probability value of the appearance of the first image-side event on the tenth. S1 is correct in determining the value of a probability, but it is not appropriate to write down the function of the probability in question. On the answer sheet, S1 writes only the value of the P(c) = $\frac{2}{59.049}$. The misconception that S1 does is not write down the probability function in the general form that it should $p(x) = \left(\frac{1}{3}\right)^9 . \frac{2}{3} = \frac{2}{59.049}\ ; x = 10$.

The misconceptions found in the initial analysis of this subject were then further deepened through interviews. In addition to deepening the data obtained, this interview is also to clarify the initial findings in the analysis of the subject's written answers. Based on the results of the interview, S1 can explain again the steps in solving question number 1, namely determining in advance each probability from the appearance of the number side and the side of the image. S1 says that due to the appearance of the image side event (G), 2 times more often than the number side event (A), each of the probabilities is P(G) = $\frac{2}{3}$ and P (A) = $\frac{1}{3}$. It means already understand the main problem that will be determined the solution. The main concept of probability has been well mastered by the subject. Then, to check students' understanding of the concept of probability function, the subject is asked:

P: "Of your answer, which is the function of the probability?"
S1: '(shows the answer on its worksheet)'



$$= \frac{2}{59049}$$

P: " Let's explain again how to get the answer"
S1: "from the written question Image probability P(G) is twice the chance of Numbers P(A), actually, I already understand how to work on it, I confused the next step, ma'am... So I multiplied the number of chances to nine, then multiplied by the equal number of chances"
P "In addition to multiplying the probability values of each of the above, do you think there are other ways to determine the function of the probability in question number 1?"
S1: ' (think for a moment).." there may be ma'am, but I kind of forgot"
P: "Can use a discrete special distribution method, do you remember?"
S1: "... (thinking)... "O.. using special distribution methods geometric discrete"

Based on the results of the interview, to describe the chances of the appearance of the first image-side events in the tenth, then S1 elaborates that on the first to ninth throws that appear in the side of the number, so according to him the chances are multiplying P (A) by 9 times then multiplied by P (G). Subject misconceptions arise in the concept of discrete probability functions, where S1 does not list probability functions based on general form or based on geometric specific distributions. S1 assumes that the value of $\frac{2}{59.049}$ is a function of the probability of the appearance of the first image-side event in the tenth. The answer to S1 is not wrong, but S1 has a misconception in distinguishing between determining the value of a probability and the probability function. If S1 writes the answer at number 1 with $p(x) = \left(1 - \frac{2}{3}\right)^{x-1} . \frac{2}{3}; x = 1,2,3, \dots \dots ,10 \; atau \; p(x) = \frac{2}{59.049}\; ; x = 10$ would be more appropriate.

To see other misconceptions made by students in solving questions about discrete random variables, analysis and interviews were conducted with Subject 2. Here's the answer is given by S2.

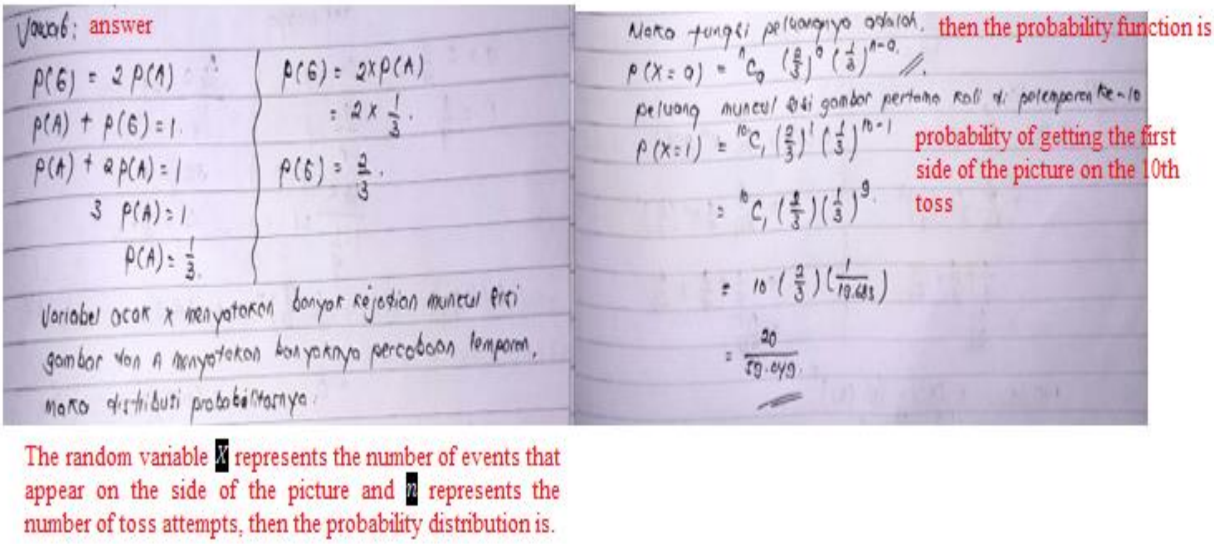


Figure 2. Answer S2

Same with S1, based on figure 2, S2 can determine the value of each probability from the occurrence of the number side and side of the image, i.e. $(G) = \frac{2}{3}$ and $P\ (A) = \frac{1}{3}$. Unlike S1, S2 already knows that what is asked about the question is the probability function, so that S2 does not just write down the value of the probability. S2 tries to describe its probability attribution by writing down the probability function from the specific first so that the general form of the probability function is obtained. Based on the description done by S2, the probability function is shaped like a binomial discrete specific probability distribution formula, that is $P(X = a) = C_a^n \left(\frac{2}{3}\right)^a \left(\frac{1}{3}\right)^{n-a}$. As a result, S2 is mistaken in determining the probability function used for the problem. The misconceptions found in the initial analysis of this subject were then further deepened through interviews. Here's an excerpt of the interview with S2.

P: "From your answer, how to determine the function of probability?"
S1: "First time I look up the value of each probability for the appearance of numbers and images, then to find the probability function, then I use a combination formula".
P: "what is the reason?"
S1: "Because the question of the tenth throwing probability function, if not wrong using a special distribution that is binomial, right not Ma'am?"
P: "If the distribution is binomial special, what are the special characteristics of?"
S1: "The throwing event was successful and failed, and the number of throws n times"
P: "The difference is the same as what geometric distribution?"
S1: "If the geometry of the probability when a successful event occurs the first time... O…, I am mistaken between binomial and geometric Ma'am.."

The misconception that S2 performs is when determining the specific distribution used, which is the binomial special distribution. Binomial distribution with geometric distribution is an equally discrete special distribution, differing only in its properties. If the geometric distribution has its experimental properties repeated several times until the success event occurs the first time, then the binomial distribution has the nature of the experiment repeated several times and determined many repetitions (Herrhyanto & Gantini, 2014).

Based on the analysis of the test results of the subject and the results of the interview analysis with the subject, the misconceptions experienced by the subject in the discrete random change material, namely in the process of determining the function of probability. S1 assumes that the probability function is equal to the probability value, whereas S2 cannot distinguish properties on geometric and binomial discrete special distributions. The findings of this study correspond to previous research that showed that some students were unable to understand random events and that their reflections were based on subjective beliefs (Gürbüz et al., 2012). Such misconceptions lead to a constant misconception in determining the function of opportunities. This will result in other misconceptions related to the probability function material, such as determining expectations and variance. Radatz classifies these types of errors in five forms, namely: 1) errors in understanding mathematical concepts, symbols, and vocabulary; 2) difficulty in processing iconic and visual representations of mathematical knowledge; 3) lack of necessary skills, facts, and concepts; 4) negative transfer caused by decoding and encoding of information; and (5) the application of irrelevant rules or strategies (Muzangwa & Chifamba, 2012). When viewed based on the type of misconception, S1 and S2 make the third type of error that is lack of skills, facts, and concepts required. S1 and S2 may forget or cannot remember related information in troubleshooting the issue.

Student misconceptions about probability will hinder the development of a correct understanding of some of the next important statistical concepts (Khazanov &Prado, 2010). Previous research has shown that individuals make mistakes when they reason about probability. Van Dijk &Zeelenberg stated that this situation was related to the fact that they were not familiar with probabilistic information and therefore had difficulty understanding it. When subjects try to use uncertain information, they tend to fiscally estimate possible events (Agus et al., 2014). Thus, the subject is said to have no conceptual understanding when solving problems do not utilize concepts that have been studied for no logical reason, or utilize some concepts related but failed in their implementation, or there is a misconception and misrepresentation of the concepts related to the resolution of the given problem (Widada, 2017). A random variable is a fundamental concept in probability theory that is not intuitive, so some students develop misconceptions about it. One common misconception is that a random variable X is any real-valued function on the sample space. Students can overcome this misconception by learning the most important characteristic of the random variable (Kachapova & Kachapov, 2012).

In addition, some mathematical concepts in different content areas are very difficult to understand. Even teachers can sometimes have misconceptions about the material (Burgoon et al., 2011). For them, this may be a very abstract concept, an intuitive or quite complex counterpoint. Therefore, changing the framework of teachers is the key to improving the teaching of mathematics for the better (Skott, 2019). Thus, as educators, a teacher and lecturer need to know the possible reasons behind the misconceptions and take precautions to provide a more efficient learning environment (Ojose, 2015); be able to recognize and face common mistakes in students probabilistic thinking (Garfield & Ahlgren, 2020). Then, if it can determine and eliminate student misconceptions the lecturer will better understand the background and perception of students about the course and determine the right learning method for them (Ay, 2017).

CONCLUSION

Probability theory is an important course for mathematics education students. This study showed some misconceptions made by students in solving the problem of probability theory, especially the function of the probability of one discrete random variable. The discussion in this study shows that students do not make use of concepts that have been studied well, or can utilize the concept but are not precise in implementing it. Advice given to lecturers or other researchers, it is necessary to study the types of misconceptions experienced by students, to provide certain treatment according to the type of misconception.

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