ISSN 2964-4291
https://proceeding.unnes.ac.id/index.php/iset

Students' Mathematical Construction Process in Solving Geometry Problems on Minimum Competency Assessment Questions

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Abstract. The purpose of this study was to describe the students' mathematical construction process in solving geometry problems on MCA questions based on APOS theory. This research is a type of exploratory descriptive qualitative research conducted at SMK PGRI 1 Kudus in the academic year 2021/2022. The research subjects were 30 students of class XI. The research instrument used is interview guide, documentation, and numeracy literacy test questions. APOS theory is used to analyze the construction process of students in solving geometry problems in numeracy literacy questions. The data analysis technique used is data reduction, data presentation, and conclusion drawing data. The results showed that at the action stage, the subject was able to correctly identify numeracy literacy problems. Furthermore, in the process stage, the subject can find a relationship between known problems and concepts that have been understood previously. This is done by recalling trigonometric formulas and comparison formulas. At the object stage, the subject can determine the height of the gazebo and the area of the gazebo correctly. At the schema stage, the subject can generalize that the height and area of the gazebo roof can be found by drawing a gazebo sketch to facilitate the completion process. The subject uses the concept of trigonometric formulas and comparative formulas to find the solution. Subjects can find answers correctly and completely in solving geometry problems on MCA questions.

Key words: geometry; mathematical construction process; minimum competency assessment.

How to Cite: Bintoro, H. S., Sukestiyarno, Y. L., Mulyono, M., Walid, W. (2021). Students' Mathematical Construction Process in Solving Geometry Problems on Minimum Competency Assessment Questions. *ISET: International Conference on Science, Education and Technology, 7*(1), 246-252.

INTRODUCTION

Minimum competency assessment (MCA) is a competency assessment that is very much needed by students to study learning materials. The components in the MCA include literacy and numeracy skills. Numerical literacy skills help students solve problems in everyday life. Research result (Nahdi et al., 2020) shows that students who have high numeracy literacy give the best contribution to mathematical problem-solving abilities. The minimum competency assessment is part of the national assessment to comprehensively capture the quality of the process and learning outcomes of primary and secondary education units throughout Indonesia (Prime, 2021). Minimum competency assessment is an assessment of basic competencies needed by all students to be able to develop their capacity and participate positively (Kemendikbud, 2017) (Aunurrahman, 2020).

Mathematical ability is a must to be mastered by all levels of society (Bintoro et al., 2021). In the MCA questions, the mathematics material is very dominant. One of the materials in MCA and considered difficult is geometry. In line with (Bintoro & Sumaji, 2021) that one of the main subjects of mathematics that is difficult for students to understand is geometry. Geometry material presents shapes, sizes, images, and the nature of space. Geometry ability is divided into several levels according to Van Hiele, starting from the visualization level, analysis level, informal deduction level, deduction level, and rigor level. Some research results, that high school students are still at the analytical level (Wang & Kinzel, 2014) (Sinclair et al., 2016). Geometry is considered difficult because it contains several complex things, which involve the mathematics of shapes, sizes, relative positions of lines and planes, and the properties of objects in 2 and 3-dimensional space, including reflection, dilation, and transformation of contraction in planes and spaces, and representation of coordinates. objects such as lines, planes, and conic sections (Harel, 2019).

Mathematical thinking processes in solving geometry problems on MCA questions need to be constructed to obtain students' thinking flow in solving problems correctly. One of these thinking construction processes uses the Action-Process-Object-Scheme (APOS) theoretical framework. The theory states that the structure of thinking is traced to the framework of mental structures (actions, processes, objects, and schemas) and mental mechanisms (interiorization, processes, reversal, encapsulation, de-encapsulation, and thematization) in constructing a concept (Dubinsky & Mcdonald, 2001) (Arnon et al., 2014). APOS theory says that the teaching of mathematics should be based on helping students to use the mental structures they already have and to build new, stronger structures, to solve the growing mathematics (Arnawa et al., 2019).

Based on the results of previous studies, no theory discusses the relationship between numeracy literacy skills in MCA and students' thinking processes that are studied based on the APOS theory in solving problems. In this study, the researcher gave the MCA geometry problem to students, then the construction of thinking was studied with the APOS theory. This study aims to reconstruct students' mathematical thinking in solving geometry problems on MCA based on APOS theory. This research is expected to help students find ideas, develop reasoning, understanding concepts in solving MCA problems in geometry.

METHOD

This type of research is qualitative with the exploratory method. The exploratory method is used to find subjects who meet the criteria of using complete mental structures and mental mechanisms in constructing new knowledge and are communicative in providing information about the work process. This study describes the mathematical thinking process in reconstructing MCA questions on geometry for class XI students of Vocational High School PGRI Kudus. The research steps: (1) give written MCA assignments, (2) conduct interviews to confirm student answers to obtain information that has not been obtained from the results of written MCA assignments.

This research was conducted at the PGRI 1 Kudus Vocational High School majoring in Class XI TKKR. The subjects of this study were 30 students. Taking into account the variety of answers and the uniqueness of the answers, four people were selected to participate in in-depth interviews. Next, the researcher chose one of four students to participate in an interview discussing the mathematical thinking process that occurred. The reason for selecting the subject is based on the fulfillment of the criteria using a complete mental structure and mental mechanism.

The data collection instruments used in this study were MCA questions on geometry material and in-depth interviews. Data analysis techniques in this study are data reduction, data presentation, and concluding. Researchers analyzed written answers and interview results to obtain data about the mathematical thinking process in reconstructing students' new knowledge based on APOS theory. The steps in data analysis include: transcribing data, reducing data, compiling data and grouping by coding, and making conclusions.

RESULTS AND DISCUSSION

The following is an explanation of the results of research related to the mathematical construction process in solving geometry problems on MCA questions based on APOS theory. The subject consists of one student who fulfills the four mental structures and the six mental mechanisms in APOS theory. Subjects were given MCA questions on geometry material and then conducted in-depth interviews to trace the flow of students' mathematical thinking processes in reconstructing geometric concepts on MCA questions.

In the mental structure of action, the subject reads the questions carefully. The subject did the interiorization by identifying the components of the problem, including the shape and size of the gazebo and the shape and size of the roof made of zinc. The subject began to think about how high the gazebo roof is and the area of zinc needed as a gazebo roof.

In the mental structure of the process, the subject uses a mental coordination mechanism by making a complete sketch of the gazebo, starting from the bottom to the roof. The sketches are all given sizes according to what he knows. The subject then proceeds to process the relationship that is owned by a right triangle. The following are the results of the assignment at the coordination stage to determine the size of the roof frame.

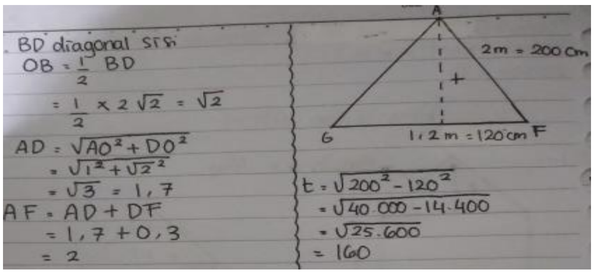


Figure 1. Coordination Results in the Form of Written Assignments

The subject's verbal form on the mental coordination mechanism can be traced through interviews. The following is an excerpt of an interview with the subject.

R; What do you think about determining the area of zinc that will be used to make the roof?
S: by finding the height of the roof and finding all the lengths of the triangular roof truss.

In Figure 1 and excerpts of the interview, it shows the subject coordinating by making more specific sketches to find the area of each side of the gazebo roof which is in the form of an isosceles triangle. The subject looks for all sizes on the gazebo roof, namely the length of the bottom and side frames and the height of the gazebo roof. The subject tries to find the length of the entire frame in various ways.

In the mental structure of the process, the subject also performs a mental reversal mechanism by recalling previous knowledge about the concept of comparison and the Pythagorean formula. The following are the results of the assignment at the reversal stage to determine all sizes of the roof frame.

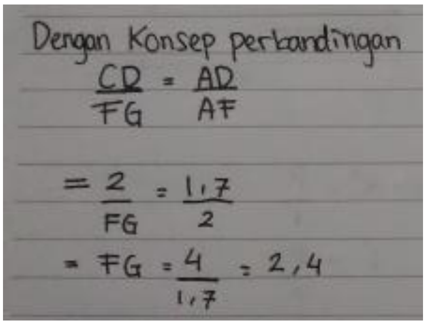


Figure 2. Reversal Results in the Form of Written Assignments

The subject's verbal form related to the mental reversal mechanism can be traced through interviews. The following is an excerpt of an interview with the subject.

R; How do you find the lengths of all roof truss sizes?
S: On the triangular base, I searched using the ratio formula and the height of the triangle on the gazebo roof using the Pythagorean formula.

In Figure 2 and excerpts of the interview, it is shown that the subject performed a mental reversal mechanism by recalling the Pythagorean formula and the formula for the comparison of two similar triangles. The subject uses a comparison formula to find the triangular base which is one side of the gazebo roof. The roof of the gazebo consists of four isosceles triangles. To find the area of the roof of the gazebo, simply look for the area of one side and then multiply by four. To find the height of the triangle on the roof of the gazebo using the Pythagorean formula.

In the mental structure of the object, the subject performs the mental mechanism of de-encapsulation first. The subject describes or disassembles the knowledge that has been previously owned about the shape and size of one side of the roof on the gazebo. The following are the results of the assignment at the de-encapsulation stage to determine the height of the roof.

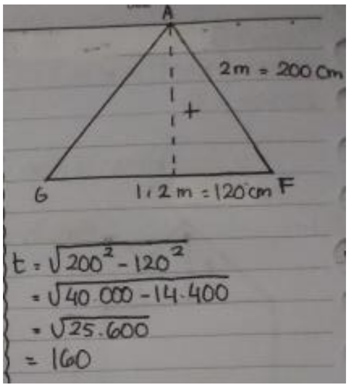


Figure 3. De-encapsulation results in the form of a written task

The subject's verbal form related to the mental mechanism of de-encapsulation can be traced through interviews. The following is an excerpt of an interview with the subject.

R; How do you find the height of the roof truss?
S: By finding the length of the base of the triangle, then finding the height using the base of the triangle and the side of the triangle which is the hypotenuse.

In Figure 3 and excerpts of the interview, it is shown that the subject performed a mental de-encapsulation mechanism by deciphering and disassembling the ratio formula of similar triangles correctly. The subject disassembles previous knowledge about the principle of comparative formulas. The comparison formula is used to find the base of the triangle, so it will be easier to find the height of the triangle which is the shape of the roof of the gazebo. Subjects can determine the height of the gazebo roof correctly.

In the mental structure of the object, the subject encapsulates the process by packaging the process into an object in written or oral form. Encapsulation in the form of a written task is to calculate the height of the roof using the tangent trigonometric ratio formula. Here is the encapsulation in the form of a written task.

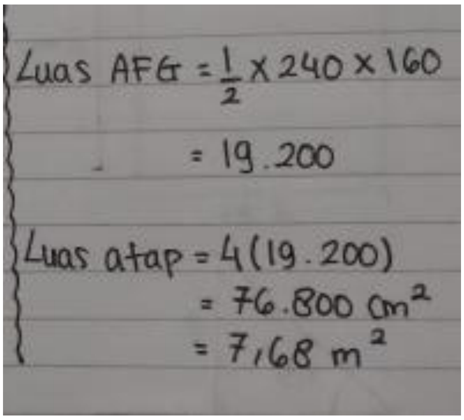


Figure 4. Encapsulation Results in the Form of Written Assignments

The subject's verbal form related to the mental encapsulation mechanism can be traced through interviews. The following is an excerpt of an interview with the subject.

R; What do you think about determining the roof area of the gazebo?
S: The roof of the gazebo consists of four equal triangles, so it is enough to find only one triangle area. After meeting one area of the triangle, the area is multiplied by four, so that the overall roof area of the gazebo will be found.

Figure 4 and interview excerpts, shows that the subject performs a mental encapsulation mechanism by finding the area of an isosceles triangle with a known base and height. To facilitate the calculation, the subject makes the size in cm so that the number is not in decimal form. The area of the roof is found by multiplying the area of the triangle four times. The roof area is then converted back into meters according to the request. The subject answered correctly the area of zinc used for the gazebo roof correctly.

In the mental structure of the schema, the subject did thematization verbally by generalizing that to find the area of the gazebo roof by finding the area of each side. The subject can determine the roof of the gazebo in the form of a rectangular pyramid with a square base. The subject translates the MCA questions by representing them into a sketch to make it easier to determine all sizes on the gazebo roof. The subject started by determining the length of the base and the height of the triangle on the roof of the gazebo. This is used to find the side area of the gazebo roof which is triangular. The subject can find one side of the gazebo roof area so that the area of zinc needed for the gazebo roof is also found correctly. This shows that the subject carries out the thought process by using reasoning correctly.

R: Once you can determine how much zinc is needed to make a roof, what do you know about the broad concept? Tell me!?
S: In my opinion, the concept of area is an area or area on an object

This shows that the subject does thematization in oral form by making generalizations. The following is an analysis in reconstructing students' mathematical thinking in solving geometry problems in MCA theory-based MCA questions shown in Figure 5.

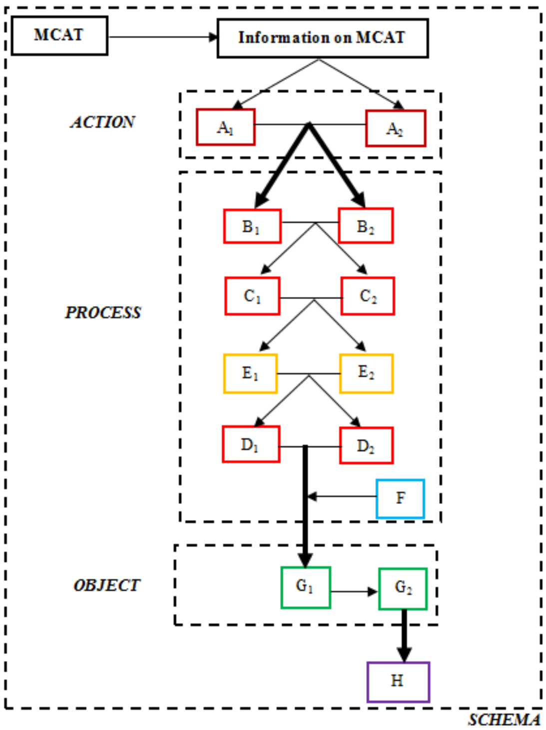
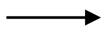


Figure 5. Analysis of the mathematical construction process based on the APOS theory

Table 1. Description of the code in Figure 5

Graphic Code	Information	Graphic Code	Information
MCAT	MCA Test	G1	Determine the area of the triangle on the sloping side of the gazebo roof
A1	Identify the length of the base	G2	Determine the area of zinc needed to cover all gazebo roofs
A2	Identify the size of the height of the triangle	H	Generalize that the amount of zinc needed to make a roof is the surface area of a rectangular pyramid without a base
B1	Making a gazebo sketch		MCA Test
B2	Sketch the roof of the gazebo		Interiorization
C1	Determine the length of the base of the triangle		Coordination
C2	Determine the height of the triangle		reversal
D1	Using the comparison formula of similar triangles to find the length of the gazebo roof base		Encapsulation
D2	Using the Pythagorean formula to determine the height of the triangle which is the sloping side of the gazebo roof		De-encapsulation
E1	Remember the formula for the ratio of similar triangles		Thematization
E2	Remembering the Pythagorean formula		Performing mental mechanisms
F	Describe and disassemble the principle of the ratio formula of similar triangles		Switch to another mental structure

Based on Figure 5 and Table 1, shows the mental structure of the subject's action to write down the components of the problem in the MCA questions. The subject identified in detail the information contained in the MCA questions. The subject uses a mental interiorization mechanism, that is, when the individual repeats and reflects on an action, the action may be internalized into the mental process (Borji et al., 2018). The subject can understand the MCA question well. Subjects have good literacy skills so they can translate questions into mathematical form. Subjects also have good numeracy skills so that they can calculate problem-solving correctly. The teacher's role in learning influences students' literacy skills. At SMK PGRI 1 Kudus, learning has implemented thematic learning with more reasoning questions. This learning makes students' thinking flow better. This is in line with an opinion (Sayekti et al., 2021) that teachers with a good understanding of numeracy literacy and doing good learning affect students' numeracy literacy skills in solving MCA problems. APOS theory considers it necessary to use teacher experience to build models that describe how students construct knowledge in a given mathematical domain (Bosch et al., 2017).

In the mental structure of the process, the subject represents it in the form of a sketch. The gazebo is drawn to an exact known size. Subjects have good conceptual skills so they can find ways to calculate the area of zinc used for the gazebo roof. The subject started by calculating the length of the base and the height of the gazebo roof. Subjects use prior knowledge of the comparison of similar triangles and the Pythagorean theorem to solve them. The subject can determine the way to determine the solution, then the subject performs a mental encapsulation mechanism. When a student considers a process as a whole and performs and constructs knowledge, the process has been encapsulated into an object of mental construction (García-Martínez & Parraguez, 2017). At the beginning of the construction process, the subject was able to understand contextual problems. Then proceed with a good understanding of the concept so that the subject can find ways to complete the solution. This is in line with an opinion (Suwandayani et al., 2020) that learning by providing contextual problems and planting concepts will increase students' numeracy literacy skills.

In the mental structure of the object, the subject can find the right answer. Subjects can remember previous knowledge and can disassemble their knowledge to solve MCA problems. Through a clear and correct sketch of the image, the subject can find the correct answer. MCA questions are given specifically to geometric problems so that representations with sketches make it easier for students to solve them. Subjects can think to relate contextual problems to mathematical problems by using image representations. This is in line with (Ratnaningsih & Hidayat, 2021) that students who have developed thinking habits and are proficient in understanding contextual problems and combining different representations will provide explanations and communication with appropriate arguments. At this stage, the subject performs a mental de-encapsulation mechanism to get the right solution. The application of the de-encapsulation mechanism allows an individual to return to the process that gave rise to the object so that the individual can solve the problem appropriately (Kazunga & Bansilal, 2017).

In the mental structure of the schema, the subject uses a mental thematization mechanism, namely to generalize that the amount of zinc needed in making the roof of the gazebo is the area of the roof of the gazebo. The subject can identify that the roof of the gazebo is a rectangular pyramid. The area of the tin roof is the area of the four upright sides. The subject can conclude that the area is an area or area in a flat shape. In APOS theory, schema thematization is a mechanism associated with transforming schemas into objects. When students can perform actions or processes on a given schema such as, for example, operate it, decompose it to use its components, and recompose it when the conditions of the problem that can be worked with it change or compare it with others (Fuentealba et al., 2017).

CONCLUSION

Subjects reconstructed mathematical thinking of geometric problems on MCA problems using all four mental structures and six mental mechanisms from the APOS theoretical framework. Subjects can understand MCA questions that contain numeracy literacy skills. The subject can understand well what is conveyed in the question. Literacy problems can be represented in a mathematical form and the subject can solve numeracy problems correctly. At the action stage, the subject can correctly identify numeracy literacy problems. Furthermore, in the process stage, the subject can find a relationship between known problems and concepts that have been understood previously. This is done by recalling trigonometric formulas and comparison formulas. At the object stage, the subject can determine the height of the gazebo and the area of the gazebo correctly. At the schema stage, the subject can generalize that the height and area of the gazebo roof can be found by drawing a gazebo sketch to facilitate the completion process. The subject uses the concept of trigonometric formulas and comparative formulas to find the solution. Subjects can find answers correctly and completely in solving geometry problems on MCA questions.

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