

Mathematical Reasoning Ability Analysis of Vocational High School Students in The Differentiated Treffinger Learning Model

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Abstract. This study aims to investigate, compare, and describe comprehensively the achievement of mathematical reasoning abilities and to find out whether there is an effect of the differentiated treffinger learning model on the mathematical reasoning abilities of class X students of SMK N 1 Demak. The research sample was class X AKL 1 with 36 students as the experimental class and class X AKL 2 with 36 students as the control class. Then as many as 3 experimental class students were analyzed in depth, namely students with high, medium and low responses. This research combine of quantitative and qualitative methods. From the results of the Study it was concluded that the treffinger learning model was differentiated of students' mathematical reasoning abilities in the experimental class better than the control class. The results of the study show the selected research subjects all experienced an increase in their mathematical reasoning abilities with different levels of improvement. This result strengthens the quantitative data above. Respondents in the high response category tend to be able to analyze, generalize, justify and recognize how these solutions can be implemented. Respondents with moderate category responses tend to be able to analyze. Meanwhile, respondents with low category responses tended to be unable to fully interpret the information.

INTRODUCTION

Learning mathematics at school, it should not only convey subject-related material. Students must be equipped with mathematical abilities as a provision for them to face competition in the world of work and the industrial world as well as in dealing with everyday problems. In vocational schools, mathematics is very important to improve students' competence in their profession in the future (Ozdemir & Onder-Ozdemir, 2021; Bakker, 2014). Mathematics is reasoning (Ross, 1998; Steen, 1999; Arnesen *et al.* 2019). If students' reasoning abilities do not develop, then mathematics will only become a matter of following a series of procedures and imitating examples without thinking about reasonable reasons, thus mathematical reasoning in the mathematics curriculum is very important.

Reasoning is the process of achieving rational thinking by taking into account all factors. Students' understanding of mathematics is possible for the development of reasoning skills (Gonc *et al.*, 2017). Mathematical reasoning is a core ability in human intelligence (Saxton *et al.*, 2019; O'Neill, 2019). Mathematical reasoning goes beyond real-world problem solving and includes making judgments about social problems that can be handled mathematically. It also includes making judgments about the validity of the information by considering the quantitative and logical implications. Mathematical reasoning ability is the act of analyzing, generalizing, and justifying (Loong *et al.*, 2018). Mathematical reasoning ability is the ability to analyze, generalize, and justify (Australian Academy of science, 2018). Thus, the indicators of mathematical reasoning used in this study are analysis, generalization, and justification.

Students' mathematical reasoning ability is still very low (Rosnawati, 2013). Based on research and surveys, low mathematical reasoning and problem-solving abilities are one of the reasons why students' mathematics achievement is still low. The times are changing rapidly and the problems that must be faced are increasingly complex. Therefore,

to overcome these problems, we need a way to solve a problem and produce the most appropriate solution. What needs to be done to overcome this is to pay attention to important facts that exist in the surrounding environment and then come up with various ideas and choose the right solution to then implement it in practice.

The most dominant characteristic in the Treffinger learning model is its effort to integrate the cognitive and affective dimensions of students to look for solutions that will be taken to solve problems. This means that students given the freedom to be creative in solving their own problems in the ways they want. The teacher's task is to guide students so that the directions taken by these students do not get out of trouble (Huda, 2013).

LITERATURE REVIEW

Alhaddad et al.(2015) found an increase in students' communication skills in learning mathematics using the Treffinger model was higher than students who were taught using conventional models. Hidayati, K., Nandini, N. K. S., & Adnan, (2021) concluded that treffinger learning was effective in increasing the creativity and mathematics achievement of trigonometry gifted students.

In the Treffinger learning model, students are encouraged to solve problems creatively. This process begins with observing and understanding important facts in the surrounding environment. From this observation, students then develop various ideas and concepts, then choose the most appropriate solution to then implement in real terms. The most prominent feature of this model is the integration of students' cognitive (thinking) and affective (feelings/emotions) dimensions. The goal is to help students find a problem-solving approach that suits their own way. In other words, students are given complete freedom to be creative in solving problems, choosing the method they want. The role of the teacher here is as a guide, ensuring that the approach taken by students remains relevant to the core of the problem (Huda, 2013).

According to Treffinger, as quoted by Huda (2013:321), this learning model has three main components that are detailed into six stages. 1) Understanding the Challenge. This first stage focuses on understanding the problem. Determining Goals: The teacher begins by explaining what students must achieve in learning. This is like setting a target at the beginning of the journey. Digging Data: Next, the teacher will present relevant phenomena or data, usually something interesting and can trigger students' curiosity. This is similar to giving initial instructions. Formulating the Problem: After the data is presented, students are given the opportunity to identify and formulate the problems they find. Here, students begin to define what they need to solve. 2) Generating Ideas. This second stage is about generating ideas. Generating Ideas: Students are given time and space to express various ideas or solutions that cross their minds. The teacher plays a guiding role so that students can agree on alternative solutions that will be tested. This is the brainstorming phase where all ideas are valued. 3) Preparing for Action. This final stage is about implementation and evaluation. Developing Solutions: The teacher encourages students to gather additional information, conduct experiments, and seek explanations that support their ideas. This stage turns ideas into concrete plans. Building Acceptance: Once a solution is obtained, the teacher will check and evaluate the solution. It doesn't stop there, the teacher will also provide new, more complex problems so that students can apply the solutions they have obtained to different contexts. This helps strengthen students' understanding and skills.

Differentiated learning is the teacher's attempt to adapt the learning process in the classroom to meet the individual learning needs of students. According Tomlinson, (2001) in a class that implements differentiated learning, a teacher makes consistent efforts to respond to students' learning needs. (Koeze, 2007) found that differentiated learning has a significant impact on increasing student achievement and learning motivation. Hassanein, (2016) confirm the effectiveness of using differentiating learning strategies in achieving four grades and developing creative, critical thinking and communication skills.

METHODOLOGY

Research Design

This type of research is mixed research. According to Creswell, (2014) this mixed methods research assumes that collecting various types of data that are considered the best can provide a thorough understanding of the problem under study.

The reasoning ability test instrument used a description test instrument compiled by researchers based on aspects of reasoning ability. Before testing the reasoning ability test instrument, the instrument was tried out to find out the reliability, validity, level of difficulty and discriminating power. In addition, it was also validated by three expert validators. Test items fulfilled the requirements used while those that did not meet the requirements were revised or not used at all.

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Sample and Data Collection

The population in this study were all students at SMK Negeri 1 Demak. While the sample is class X AKL 1 SMK Negeri 1 Demak for the 2022-2023 academic year, namely 36 students. Data collection through questionnaires, tests of reasoning abilities, observations, interviews and documentary studies.

Analyzing of Data

The data analysis technique used in this research was started by analyzing the validity of teaching materials and research instruments, then compared between the experimental class and the control class which were previously tested for homogeneity and normality of the data, then using the qualitative method.

Quantitative and qualitative data analysis were carried out by comparing the quantitative data resulting from quantitative research and the qualitative data obtained at the same time. Through this data analysis, information can be obtained whether the two data complement each other, expand, deepen or even contradict each other. If two groups of data are found contradictorily, then the credibility of the qualitative research data is tested again until the truth of the data found, by extending observations, increasing persistence, conducting triangulation, analyzing negative cases and checking members. Furthermore, the results of the research used are the results of qualitative research that are correct / certain that have been tested for credibility (Sugiyono, 2013).

FINDINGS / RESULTS

Differentiated Treffinger Results and Learning are as follows:

Tabel 1 Analysis of student activity in differentiated treffinger learning

Total students	six meetings						Post Test
	1	2	3	4	5	6	
36	60%	72%	75%	80%	86%	91%	Test

The results of observations of student learning activities in receiving learning with the treffinger model differed after the pretest or at the first meeting, 60% were still included in the good category. This continued until the 3rd meeting. Meanwhile, the 4th and 5th meetings were included in the very good category, namely the percentages of 80% and 86%. and the 6th meeting was included in the very good category, namely with a percentage of 91%. The average value almost increases at each meeting.

Quantitative analysis of mathematical reasoning abilities is obtained according to table 2 below,

Table 2.
Quantitative Analysis Results

Test	hypothesis	Sig. Value	Results
Normality test	h0 : normally distributed h1 : not normally distributed	0.200 (A) 0.135 (B)	H0 is accepted
Homogeneity Test	h0 : homogeneous data h1 : data is not homogeneous	0,235	H0 is accepted
One sample T-Test	h0 : no significant difference between Grade A and B math reasoning h1 : There is a significant difference between Class A and B mathematics reasoning	0,000	H0 is accepted

Based on the results of this analysis it can be stated that class A (Control) and B (Experimental) are normally distributed with a significance value of 0.200 (A) and 0.135 (B), the data is called normal if the significance value is greater than 0.05. So is the significance value of homogeneity with the acquisition of 0.235. Therefore it can be said to be homogeneous. The one sample t-test is a sample test that aims to find out whether there are differences in the reasoning abilities of students using differentiated Treffinger and conventional learning. In this analysis the value of the one sample t-test is less than 0.05 so that it can be said that there is a significant difference between the mathematical reasoning abilities of class A (Control) and B (Experimental) students.

In the third column, $\text{sig} = 0.235 = 23.5\% > 5\%$, meaning that the two groups has the same (homogeneous) variance. In t column 6, $\text{sig} = 0.000 = 0\% < 5\%$, meaning that the experimental group's learning average is different from the control group. It can be seen from the output group statistics that the average for the experimental class is 75.03, which is far greater than the average for the control class, 49.92. This shows that the mathematical reasoning abilities of the experimental class students are better than the control class' mathematical reasoning abilities. It can be concluded that learning with a differentiated treffinger model which is able to provide changes in increasing students' mathematical reasoning abilities.

Based on the posttest results of the experimental class students above, it shows that students are able to reason on mathematical statements in questions where students are able to present to analyze, generalize an justifycognize how these solutions can be implemented. Unlikely the case with the posttest results of control class students, where students were only able to present analyze.

DISCUSSION

① Misal : buku fiksi = x
Bulpen = y
Penghapus = z

Diket : $8x + 4y + 3z = 62.000$ (*)
 $5x + 8y = 57.000$ (a)
 $3x + 7z = 17.000$ (b)

Jawab : * Toko Rejeki

Eliminasi (1) dan (a)

$$\begin{array}{r} 8x + 4y + 3z = 62.000 \\ - 5x + 8y = 57.000 \\ \hline 3x - 4y - 3z = 5.000 \end{array}$$

Eliminasi (2) dan (b)

$$\begin{array}{r} 3x - 4y - 3z = 5.000 \\ - 3x + 7z = 17.000 \\ \hline -4y + 4z = -12.000 \end{array}$$

Substitusi (c) ke (a)

$$\begin{array}{r} 5x + 8y = 57.000 \\ - 5000 + 8y = 57.000 \\ 8y = 62.000 \\ y = 7.750 \end{array}$$

Substitusi (c) ke (b)

$$\begin{array}{r} 3x + 7z = 17.000 \\ - 3(5000) + 7z = 17.000 \\ -15.000 + 7z = 17.000 \\ 7z = 32.000 \\ z = 4.571 \end{array}$$

Jadi, harga buku fiksi di toko Rejeki = 5000
harga bulpen " = 4000
harga penghapus " = 2000

Jadi, Bu Sri menarik kesimpulan bahwa harga buku fiksi lebih mahal Toko Makmur daripada Toko Rejeki, harga bulpen lebih murah Toko Makmur daripada Toko Rejeki, jika membeli ketiganya maka harga lebih murah Toko Makmur.

* Toko Makmur

Eliminasi (1) dan (a)

$$\begin{array}{r} 8x + 4y + 3z = 62.000 \\ - 5x + 8y = 57.000 \\ \hline 3x - 4y - 3z = 5.000 \end{array}$$

Eliminasi (2) dan (b)

$$\begin{array}{r} 3x - 4y - 3z = 5.000 \\ - 3x + 7z = 17.000 \\ \hline -4y + 4z = -12.000 \end{array}$$

Substitusi (3) ke (2)

$$\begin{array}{r} 2x + 3y = 22.500 \\ - 2x + 10.500 = 22.500 \\ 10.500 = 0 \end{array}$$

Substitusi (3) ke (1)

$$\begin{array}{r} 8x + 4y + 3z = 62.000 \\ - 8(6000) + 4(3500) + 3(2000) = 62.000 \\ -48.000 + 14.000 + 6.000 = 62.000 \\ -28.000 = 62.000 \end{array}$$

Jadi, harga buku fiksi di toko Makmur = 6000
Bulpen = 3.500
Penghapus = 2000

Figure 1. The results of student work on students with high category responses

Respondents in the high response category tend to be able to analyze, generalize, justify, and recognize how these solutions can be implemented.

Increasing students' reasoning abilities here can occur because the treffinger learning model has the most dominant characteristics in this case its efforts to integrate cognitive (reasoning abilities) and affective (student responses in learning) dimensions of students to find directions for completion that will be taken to solve the problem. This means that students given the freedom to be creative in solving their own problems in the ways they want. The teacher's task is to guide students so that the directions taken by these students do not get out of trouble. So that it can improve the aspect of mathematical reasoning ability.

This is in accordance with the research results of Retnowati & Murtiyasa (2013) which state that the use of the treffinger model in learning mathematics can improve students' understanding of mathematical concepts and dispositions. Alhaddad et al., (2015) found that the increase in students' communication skills in learning mathematics using the treffinger model which was higher than students taught using conventional models. Hidayati, K., Nandini, N. K. S., & Adnan, (2021) concluded that treffinger learning was effective in increasing the creativity and mathematics achievement of gifted students in trigonometry.

CONCLUSION

Based on the overall discussion above, learning mathematics with the differentiated Treffinger learning model yielded results: learning with the differentiated Treffinger model was able to provide changes in improving students' mathematical reasoning abilities. According to the results of qualitative data analysis, the selected research subjects all experienced an increase in their mathematical reasoning abilities with different levels of improvement. This result strengthens the quantitative data above. Respondents in the high response category tend to be able to analyze, generalize, justify and recognize how these solutions can be implemented. Respondents with moderate category responses tend to be able to analyze. Meanwhile, respondents with low category responses tended to be unable to fully analyze.

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