ISSN 2964-4291
https://proceeding.unnes.ac.id/index.php/iset

The Analysis of Problem Solving Ability Viewed from Intuition in Integral Calculus Course

Eko Andy Purnomo, Yohanes Leonardus Sukestiyarno*, Iwan Junaedi, Arief Agoestanto

Universitas Negeri Semarang, Semarang, Indonesia
*Corresponding Author: sukestiyarno@mail.unnes.ac.id

Abstract. Students need to engage in analytical and logical thought processes, as well as the construction of mathematical knowledge and ideas, to solve problems. What these students are doing is an example of intuitive cognition. From what we can tell, many students, especially those taking Integral Calculus, do not fully use their mental capacities when attempting to solve issues. This research aimed to identify the extent to which intuition is used to solve problems encountered in the study of Integral Calculus. The method of this research was a descriptive qualitative method. A total of 43 participants from the FMIPA UNIMUS Mathematics Education Study Program participated in the study. The study's findings were that the problems persisted regardless of whether the children were high, middle, or poor achievers. The instruments used in this study are the evaluation questions, the intuition surveys, and the interviewing procedures for both the problem solver and the intuitive. They used evaluation tests, observations, and in-depth interviews to triangulate their results. Data analysis entails three stages: data reduction, display, and verification. Affirmatory intuition was most common among students with high problem-solving abilities. In contrast, those with average skills utilized a mix of Affirmatory and Anticipatory intuition. On the other hand, students with limited talents relied on Anticipatory intuition rather than actual intuition. The findings suggest that when presented with a problem, pupils' first instincts are not universal. It indicates that further investigation would develop pupils' innate ability to solve problems creatively.

Key words: integral calculus; intuition; problem-solving.

How to Cite: Purnomo, E. A., Suksetiyarno, S., Junaedi, I., & Agoestanto, A. (2022). The Analysis of Problem Solving Ability Viewed from Intuition in Integral Calculus Course. *ISET: International Conference on Science, Education and Technology* (2022), 246-251.

INTRODUCTION

The main goal in learning mathematics is to be able to solve problems in everyday life. Problem-solving is the most crucial skill for students (Damayanti, & Sukestiyarno, 2014) (Erlina & Purnomo, 2020) (Sulistyaningsih et al., 2021) (Purnomo et al., 2022). It is part of the mathematics curriculum in almost all countries, including the United States (Schoenfeld, 2007), Australia (Clarke et al., 2007), the Netherlands (Doorman et al., 2007), China (Cai & Nie, 2007), France (Artigue & Houdement, 2007), Hungary (Szendrei, 2007) and English (Burkhardt & Bell, 2007). Problem-solving in mathematics is an activity to find solutions to mathematical problems by involving all knowledge (have studied concepts) and experience (have been trained and accustomed to facing or solving problems), which does not require a particular pattern of methods or strategies. the solution (Muniri, 2013). Efforts to get solutions or answers to mathematical problems differ from one student to another (Etika et al., 2016). Some students view problem-solving as challenging, while others find it easy.

In addition to requiring mental activities that are analytical and logical, the process of formulating mathematical knowledge, including building ideas to solve problems, requires other different cognitive activities. The cognitive activity in question is intuitive cognition or intuition. Sometimes, suppose the problem is unfamiliar or has nothing to do with informal knowledge. In that case, someone can solve directly, spontaneously, quickly, and irregularly the steps in solving the problem, which means this second part is classified as intuitive thinking. Based on this, it can be concluded that intuition can be used in solving problems.

One of the problems in the Integral Calculus course is that students have not been able to apply internal material to solve problems in everyday life. Based on research, students' problem-solving abilities are still low (Purnomo & Mawarsari, 2014) (Purnomo et al., 2014); (Wardono et al., 2016); (Hidayat & Irawan, 2017) ; (Yeni et al., 2020). Students have mastered the definition and integral theorem but cannot solve application problems. Students have difficulty using definitions and theorems when solving application problems. One of the causes of insufficient problem-solving ability is that students are not used to solving problems. Problem-solving activities rarely cause students not to have good intuition.

Problem-solving can run well when someone has experience solving (Greiff, S & Fischer, 2013) and does not experience cognitive barriers (Antonijević, 2016). One of the most supportive abilities in problem-solving is intuition. Intuition is a unique form of processing in information processing. Conceptually, information processing systems consist of conscious and subconscious processing. Conscious processing systems allow individuals to analyze problems intentionally and sequentially and devote their attention, while subconscious processing allows individuals to learn from experience (Kamandoko & Suherman, 2019). Intuition also has a main characteristic, speed in processing information that cannot be explained through steps. One of the impressive things about observable intuition is that the response is often correct, even though it does not seem to take time. The process underlying problem-solving intuition is pattern matching which is sharpened through repeated training and practice. Thus the intuition of problem-solving is closely related to the domain of knowledge or expertise.

In addition to Intuition, in solving mathematical problems, everyone has different ways and styles of thinking because not everyone has the same thinking ability. Everyone has unique ways of acting, expressed through perceptual and intellectual activities. Intuition plays a role in making assumptions or claims in solving mathematical problems. Also, it gives meaning or informal interpretations of specific definitions, theorems, formulas, and solving strategies (Asmara & Afifah, 2019). The use of definitions and theorems is a feature of formal cognition, while the use of formulas and strategies is a feature of algorithmic cognition. It shows that Intuition supports formal and algorithmic cognition's role in solving mathematical problems. If the student's Intuition is good, then the problem-solving ability is also good.

In this study, students' Intuition will be seen. It refers to Fischbein's opinion with indicators (Budi Usodo, 2012). Namely 1). *Affirmatory Intuition* can be defined as statements, representations, interpretations, and solutions that individuals can accept directly, self-evident, global, and intrinsically sufficient, and 2). Anticipatory intuition (anticipatory Intuition) Anticipatory intuition is an intuition that arises when someone works hard to solve a problem, but the solution is not immediately obtained (not directly). Intuition is often summarized in four phases: (i) After working on a complex problem for some time and finding no solution (preparation), (ii) the problem solver thinks about different things (incubation). (iii) After some time—hours, or even weeks—an idea suddenly appears (illumination), providing a solution or at least a significant step towards the solution of the problem; (iv) this idea must be verified. In this study, we will use the intuition stage proposed by Wallas, which consists of 4 stages: preparation, incubation, illumination, and verification (Rott et al., 2021). Intuition is widely used by students in solving problems (Muniri, 2013; Etika et al., 2016). Based on this, it is necessary to research students' problem-solving abilities regarding student intuition.

METHODS

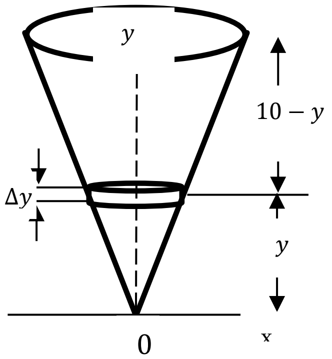
This research method is descriptive qualitative research that describes problem-solving ability in terms of intuition. The instrument used in this study tested students' problem-solving abilities and intuition. In the process of Intuition in Problem Solving, there are six stages, including 1). determine what is known, asked from the given problem, 2)—finding the unknown through the known, 3). Choose the right strategy for solving the problem, 4). It is implementing the strategy, 5). Determine the solution to the problem, and 6). Recheck the answer. Intuition in this study, there are three groups, namely Affirmatory, Anticipatory and non-intuitive.

The research subjects were students of the Mathematics Education Study Program, Faculty of Mathematics and Natural Sciences, UNIMUS, totaling 44 students. The students were students with high, medium, and low abilities. Triangulation involves intuition, namely problem-solving tests, questionnaires, and in-depth interviews. Data from tests, interviews, and observations were analyzed by content analysis, consisting of three activities: data reduction, data presentation, and conclusion (Creswell, 2012). Data analysis uses an inductive approach where conclusions are drawn from minor case investigations in detail to provide a big picture (Sukestiyarno, 2020).

RESULTS AND DISCUSSION

In the initial stage, students are given problem-solving questions. Then students are asked to solve problem-solving problems. Problem-solving problem “A tank which is conical in the shape of a vertical circle (see picture below) is full of water. If the height of the tank is 10 feet and the radius of the top circle is 4 feet. Determine the force required to pump the water:

a. Past the top edge of the tank
b. Reach 10 feet above the top of the tank



The results of solving problems can be analyzed according to the stages of problem-solving. Intuition sees each stage of the problem carried out by students: Affirmatory, Anticipatory, and not intuitive. The analysis was grouped into 3, namely high-ability students (S1), medium-group students (S2), and low-group students (S3). The results of the analysis are presented in Table 1.

Table 1. The results of Intuition Process in Problem-Solving

Intuition Process in Problem-Solving	Students/ Research Subjects		
	S1	S2	S3
Determining what is known, asked from the given problem.	Affirmatory	Affirmatory	Affirmatory
Finding the unknown through the known.	Affirmatory	Affirmatory	Anticipatory
Choosing the right strategy for solving the problem.	Affirmatory	Affirmatory	Affirmatory
Implementing the strategy.	Affirmatory	Affirmatory	Anticipatory
Determining a solution to the problem	Affirmatory	Anticipatory	Anticipatory
Recheck the answer	Anticipatory	Do not Use	Do Not Use

Based on Table 1, it can be seen that in the first stage, all students used affirmatory intuition. Students can quickly determine what is known and asked in the question. At this stage, students can directly find the answers requested. Students have determined the known information by writing down what is known, namely the height and radius of the tank. Students also write down the questions asked in the questions. It is known during in-depth interviews. Students can get answers directly and in less than 3 minutes. Based on this, in the first stage, all students can do affirmatory intuition well.

In the second stage, S1 and S2 students can determine something unknown in the problem. S1 and S2 students can determine unknown information, namely the radius of a circle with a thickness of y at the height of Δy, which is $\frac{4y}{10}$. Students find the relationship between the data and the unknown, namely the volume of the disc, namely $\Delta v = \pi(\frac{4y}{10})^2 \Delta y$ and its weight (gravity) = $\delta\pi(\frac{4y}{10})^2 \Delta y$ with δ= 62.4 (density of water). S1 and S2 students systematically and the proper steps to find something unknown. In the results of interviews, S1 and S2 students can explain clearly and well from the answers given. Doctoral students can determine the unknown but have difficulty finding it. Doctoral students use their intuition but cannot use it well. Doctoral students found something unknown in the problem but did not use the procedure correctly. At the time of the interview, doctoral students still had difficulty explaining the findings obtained from analyzing the questions.

In the third stage, all students can use Affirmatory intuition. The three students got the right strategy for solving the questions' problems. This strategy is the most appropriate strategy for solving the problem. At the interview stage, S1, S2, and S3 students can convey the strategies applied. Students can also reveal the reasons for using the chosen strategy.

At the strategy's implementation stage, high and medium-ability students can answer well using affirmatory intuition. There are two steps: looking for work for a water pump and a). past the top edge of the tank, and b). reach 10 feet above the top of the foot. The vital force is 10-y. So work $\Delta w = \delta\pi(\frac{4y}{10})^2 \Delta y\ 10 - \mathrm{y}$. So the work to pump water up to the rim of the tank is 26,138 pound-feet. Same with the previous point problem, now the water in the cone must be lifted 20-y. So the work to pump the water up to the edge of the tank is 130.69 pound-feet. Students with low abilities use Anticipatory intuition. Based on the observations and the results of the answers of low-ability subjects, it appears that there is problem-solving, is not logical, uses complicated procedures, and uses many ways by guessing and experimenting. The analysis found that the S1 and S2 students had answered the questions correctly, but in answering the many stages of problem-solving, it had not been written explicitly in the answers. It will affect the accuracy of answering questions at the strategy implementation stage.

At the stage of determining the solution to the problem, only undergraduate students use Affirmatory intuition. S1 students can answer questions systematically, and the answer is correct. At the time of the interview, S1 students can explain the answers well and quickly. S2 and S3 students use Anticipatory intuition. Students cannot explain correctly in answering questions. In the last stage, namely checking answers, undergraduate students use Anticipatory intuition. S1 students estimate alternative answers, but they are not precise and not systematic. S2 and S3 students do not use intuition in checking answers. S2 and S3 students cannot check whether the answer is correct.

This study investigated how intuitive mathematical conceptions affect students' problem-solving abilities. Solving ability will be good When intuitive conceptions lead to systematic, structured predictions compared to predictions derived from an assessment of the efficiency of using informal strategies (Gvozdic & Sander, 2018) (Van Hoof et al., 2020). To see if intuitiveness affects problem-solving, students are asked to solve problems and give reasons for written answers, revealing if they understand the strategies. The most favorable situation in learning mathematics is where students intuitively understand mathematical concepts formally. Usually, students in simple situations interpret mathematical facts by referring to concrete reality and consider formal proofs to be true.

Based on the results of the study, problem-solving ability is influenced by intuition. At the problem-solving stage, students use different intuitions according to the student's ability level (Muniri, 2013) (Etika et al., 2016) (Asmara & Afifah, 2019). High-ability students use Affirmatory intuition, and moderate-ability students use Affirmatory and Anticipatory intuition. In low ability, students use Anticipatory intuition. Students use intuition in decision-making in every stage of problem-solving. It is in line with the statement that intuition is the basis of all mathematical reasoning and decisions (Marsigit, 2006).

The results showed that undergraduate students tended to use affirmatory intuition, postgraduate students used a combination of affirmatory and anticipatory intuition, and doctoral students tended to use and did not use intuition. Based on the study's results, the students use different intuitions in solving problems. The intuition that can be used in problem-solving by using Affirmatory intuition. Through Affirmatory intuition, students' problem-solving results can be directly known, self-evident, global, and intrinsically sufficient. Therefore, intuition can be trained so that it will increase. Intuition enhancement is focused on Affirmatory intuition. This increase in intuition can improve problem-solving abilities. Therefore, lecturers must design learning activities that can train students with intuition so that an increase in intuition can be obtained, which can be used to solve problems. Learning should often be given to problem-solving using pattern stages. It aims to further explore students' intuition in solving mathematical problems.

CONCLUSION

The results showed that undergraduate students tended to use affirmatory intuition, postgraduate students used a combination of affirmatory and anticipatory intuition, and doctoral students tended to use and did not use intuition. Based on the results of the study, it can be concluded that in solving problems, students use different intuitions according to the level of students abilities.

REFERENCES

Antonijević, R. (2016). Cognitive activities in solving mathematical tasks: The role of a cognitive obstacle. *Eurasia Journal of Mathematics, Science and Technology Education*, *12*(9), 2503–2515. https://doi.org/10.12973/eurasia.2016.1306a

Artigue, M., & Houdement, C. (2007). Problem solving in France: Didactic and curricular perspectives. *ZDM - International Journal on Mathematics Education*, *39*(5–6), 365–382. https://doi.org/10.1007/s11858-007-0048-x

Asmara, B. W. A., & Afifah, D. S. N. (2019). Profil Intuisi Matematis Siswa dalam Pemecahan Masalah Matematika Ditinjau dari Gaya Kognitif Field Independent dan Field Dependent. *Jurnal Elektronik Pembelajaran Matematika*, *3*(1), 37–50. http://e-journal.unipma.ac.id/index.php/JP-LPPM/article/view/933

Burkhardt, H., & Bell, A. (2007). Problem solving in the United Kingdom. *ZDM - International Journal on Mathematics Education*, *39*(5–6), 395–403. https://doi.org/10.1007/s11858-007-0041-4

Cai, J., & Nie, B. (2007). Problem solving in Chinese mathematics education: Research and practice. *ZDM - International Journal on Mathematics Education*, *39*(5–6), 459–473. https://doi.org/10.1007/s11858-007-0042-3

Clarke, D., Goos, M., & Morony, W. (2007). Problem solving and Working Mathematically: An Australian perspective. *ZDM - International Journal on Mathematics Education*, *39*(5–6), 475–490. https://doi.org/10.1007/s11858-007-0045-0

Creswell, J. W. (2012). *Educational Research : planning, conducting, and evaluating quantitative and qualitative research* (Paul A. Smith (ed.); 4th ed.). Pearson Education.

Damayanti, & Sukestiyarno, Y. L. (2014). Meningkatkan Karakter dan Pemecahan Masalah Melalaui Pendekatan Brain-Based Learning Berbantuan Sirkuit Matematika. *Kreano: Jurnal Matematika Kreatif-Inovatif*, *5*(1), 82–90. https://doi.org/10.15294/kreano.v5i1.3281

Doorman, M., Drijvers, P., Dekker, T., van den Heuvel-Panhuizen, M., de Lange, J., & Wijers, M. (2007). Problem solving as a challenge for mathematics education in The Netherlands. *ZDM - International Journal on Mathematics Education*, *39*(5–6), 405–418. https://doi.org/10.1007/s11858-007-0043-2

Erlina & Purnomo, E. A. (2020). Implementasi Lesson Study Melalui Model Pembelajaran Problem Based Learning Materi Spltv Kelas X Iik. *AlphaMath : Journal of Mathematics Education*, *6*(1), 36–45. https://doi.org/10.30595/alphamath.v6i1.7619

Etika, E. D., Sujadi, I., & Subanti, S. (2016). Intuisi Siswa Kelas VII Smp Negeri 1 Nganjuk dalam Pemecahan Masalah Matematika Ditinjau dari Adversity Quotient (AQ). *Jurnal Elektronik Pembelajaran Matematika*, *4*(5), 563–574.

Greiff, S & Fischer, A. (2013). Measuring Complex Problem Solving: An educational application of psychological theories. *Journal of Educational Research Online*, *5*(1), 38–58. http://www.j-e-r-o.com/index.php/jero/article/download/338/160

Gvozdic, K., & Sander, E. (2018). When intuitive conceptions overshadow pedagogical content knowledge: Teachers' conceptions of students' arithmetic word problem solving strategies. *Educational Studies in Mathematics*, *98*(2), 157–175. https://doi.org/10.1007/s10649-018-9806-7

Hidayat, A., & Irawan, I. (2017). Pengembangan LKS Berbasis RME dengan Pendekatan Problem Solving untuk Memfasilitasi Kemampuan Pemecahan Masalah Matematis Siswa. *Jurnal Cendekia : Jurnal Pendidikan Matematika*. https://doi.org/10.31004/cendekia.v1i2.20

Kamandoko, & Suherman. (2019). Profil Intuisi Matematis Siswa dalam Pemecahan Masalah Matematika Ditinjau dari Gaya Kognitif Field Independent dan Field Dependent. *Kontinu: Jurnal Penelitian Didaktik Matematika*, *3*(1), 37. https://doi.org/10.30659/kontinu.3.1.37-50

Marsigit. (2006). Peran Intuisi dalam Matematika Menurut Immanuel Kant. *Prosiding Konferensi Nasional Matematika XII*, 1–9.

Muniri. (2013). Karakteristik Berpikir Intuitif Siswa dalam Menyelesaikan Masalah Matematika. *Seminar Nasional Matematika Dan Pendidikan Matematika Dengan Tema " Penguatan Peran Matematika Dan Pendidikan Matematika Untuk Indonesia Yang Lebih Baik", 9 November 2013*, 978–979.

Purnomo, E.A. & Mawarsari, V. D. (2014). Kemampuan Pemecahan Masalah Melalui Model Pembelajaran Ideal Problem Solving Berbasis Project Based Learning. *Jkpm*, *1*(1), 24–31.

Purnomo, E. A., Fathurohman, A. &, & Budiharto. (2014). Keefektifan Model Pembelajaran IDEAL Problem Solving Berbasis Maple Matakuliah Kalkulus II. *Jkpm*, *1*(2), 7–11. https://jurnal.unimus.ac.id/index.php/JPMat/article/view/1672

Purnomo, E. A., Sukestiyarno, Y. L., Junaedi, I., & Agoestanto, A. (2022). Analysis of Problem Solving Process on HOTS Test for Integral Calculus. *Mathematics Teaching Research Journal*, *14*(1), 199–214.

https://commons.hostos.cuny.edu/mtrj/wp-content/uploads/sites/30/2022/04/v14n1-Analysis-of-Problem-Solving.pdf

Rott, B., Specht, B., & Knipping, C. (2021). A descriptive phase model of problem-solving processes. In *ZDM - Mathematics Education*. https://doi.org/10.1007/s11858-021-01244-3

Schoenfeld. (2007). Problem solving in the United States, 1970–2008: research and theory, practice and politics. *Zdm*, *39*(5–6), 537–551. https://doi.org/10.1007/s11858-007-0038-z

Sukestiyarno, Y. L. (2020). *Metode Penelitian Pendidikan* (2nd ed.). Unnes Press.

Sulistyaningsih, D., Purnomo, E. A., & Purnomo. (2021). Polya's problem solving strategy in trigonometry: An analysis of students' difficulties in problem solving. *Mathematics and Statistics*, *9*(2), 127–134. https://doi.org/10.13189/ms.2021.090206

Szendrei, J. (2007). When the going gets tough, the tough gets going problem solving in Hungary, 1970-2007: Research and theory, practice and politics. *ZDM - International Journal on Mathematics Education*, *39*(5–6), 443–458. https://doi.org/10.1007/s11858-007-0037-0

Van Hoof, J., Verschaffel, L., De Neys, W., & Van Dooren, W. (2020). Intuitive errors in learners' fraction understanding: A dual-process perspective on the natural number bias. *Memory and Cognition*, *48*(7), 1171–1180. https://doi.org/10.3758/s13421-020-01045-1

Wardono, Waluya, S. B., Mariani, S., & Candra, S. D. (2016). Mathematics Literacy on Problem Based Learning with Indonesian Realistic Mathematics Education Approach Assisted E-Learning Edmodo. *Journal of Physics: Conference Series*, *693*(1). https://doi.org/10.1088/1742-6596/693/1/012014

Yeni, E. M., Wahyudin, & Herman, T. (2020). Difficulty analysis of elementary school students in mathematical problem solving in solutions. *International Journal of Scientific and Technology Research*, *9*(3), 44–47.