# How Do Students Mathematize in Solving Algebra Problems? A Case Study at SMPN 2 Balusu

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## Abstract

Algebra is an important subject in mathematics learning because it serves as a foundation for understanding higher concepts and as a tool for modeling various problems. However, many students still experience difficulties in connecting contextual problems into mathematical models. This study aims to describe the process of students' mathematization in solving algebraic problems, including the stages of understanding the problem, formulating a model, problem-solving, and reflecting. The type of research used to achieve this goal is qualitative. The research subjects consisted of three students at SMPN 2 Balusu who were selected based on their answer patterns. The results showed that each student had a different approach to solving algebraic problems. Some students were able to work sequentially, starting from understanding the problem, creating a model, and rechecking the results. There were also students who actually understood the core of the problem, but still made mistakes when creating the model, resulting in inconsistent answers. Meanwhile, some chose shortcuts by guessing numbers to get the answer.

Keywords: Algebra, Mathematize, Case Study

## 1. Introduction

Learning is essentially an active process by which individuals construct knowledge through interactions with their environment and the experiences they acquire. In an educational context, learning is not simply a process of receiving information, but rather an activity that involves thinking, reflecting, and constructing meaning (Schunk, 2012). This aligns with the constructivist view, which emphasizes that knowledge is actively constructed by students, not simply transferred from teachers. Therefore, mathematics learning should be directed at facilitating students' meaningful experiences, discoveries, and construction of concepts.

One area of mathematics that requires a high level of abstract thinking is algebra. Algebra encompasses not only symbol manipulation skills but also representation, generalization, and mathematical reasoning (Kieran, 2018). In reality, many students struggle to understand algebra because they are required to move from concrete forms to abstract formal symbols (Blanton et al., 2015). For example, when solving story problems, students often struggle when having to translate contextual situations into algebraic equations. Errors occur not only in operational procedures but also in understanding the meaning of symbols, variables, or relationships between variables.

In the context of algebra learning, problems become increasingly complex due to their abstract nature, which demands symbolic representation skills. If students are not trained in horizontal mathematization, they will have difficulty


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translating story problems into algebraic models. Conversely, if they do not master vertical mathematization, they will experience obstacles in manipulating mathematical models toward more general solutions. This condition reinforces the view that algebra often acts as a "gatekeeper" for students in pursuing higher levels of mathematics (Kaput, 2008). Therefore, investigating how students mathematize in algebra is crucial.

his gap is even more apparent in algebra learning. Kaput (2008) states that algebra serves as a gatekeeper for further mathematics learning, due to its fundamental nature in connecting arithmetic with more complex mathematical structures. Thus, students' inability to mathematize algebra is not merely a technical issue, but has broad implications for the development of their mathematical reasoning as a whole. If students cannot connect contextual phenomena with mathematical models, their skills in critical, creative, and reflective thinking will also be hampered.

One framework that can be used to analyze how students solve problems is the concept of mathematization. According to Freudenthal (1991), mathematization is the process of organizing and rearranging real-world realities or problems into mathematical forms. This process is divided into two stages: horizontal mathematization (translating contextual problems into mathematical models) and vertical mathematization (developing mathematical models through the manipulation of symbols and algebraic procedures) (Treffers, 1987).

Mathematization can be defined as a process of mathematizing a phenomenon (Wijaya, 2012). According to Vom Hofe et al. (in Prediger, 2011), “mathematizing - the step in the modeling cycle in which the transformation from a real-world situation into a mathematical problem is conducted.” Based on Vom Hofe's opinion, mathematization is defined as the transformation of a real-world situation into a mathematical problem through a cycle called the modeling cycle. There are also those who argue about mathematization as the activity of organizing mathematical ideas and concepts by using prior knowledge and skills in finding relationships (relations), regularities (regularities), and structures (structures) (De Lange, 1987). In addition, Roux argues that mathematization is the application of concepts, procedures, and methods developed in mathematics to objects of other disciplines or various other fields of knowledge (Roux, 2010). Mathematization can also be considered as a basic process used by students in solving real-life problems. This is in accordance with the PISA statement (OECD, 2009:105), "In PISA, the fundamental process that students use to solve real-life problems is referred to as mathematisation".

On the other hand, international research has emphasized the importance of mathematization within the framework of realistic mathematics education (RME) (Freudenthal, 1991; Gravemeijer, 1994). However, the cultural and educational context in Indonesia presents different challenges. Students tend to be limited in exploring various problem-solving strategies, as learning remains heavily oriented toward a single correct answer. This results in limited opportunities for students to practice mathematization independently. In other words, mathematization in the classroom often occurs top-down, through teacher direction, rather than as a result of active student construction.

A number of study show that the process of mathematization often become constraint main for students . Gravemeijer (1994) emphasized that success learning mathematics is very dependent on how student build understanding through mathematization , not just control procedure count . Meanwhile that , Wijaya (2012) found that Indonesian students still weak in build mathematical models , although they capable do calculation technical . This is show existence gap between ability procedural and capabilities conceptual student .

An in-depth analysis of students' mathematization processes in algebra is crucial not only for understanding the difficulties they experience but also for identifying critical points where pedagogical interventions can be implemented. This study seeks to fill this gap in the literature by systematically exploring how students construct, modify, and interpret mathematical models in the context of algebra. The focus is not only on the final output, namely answers, but also on the thinking processes that occur during students' mathematization, both through analysis of written answers and verbal exploration through think-alouds.

Therefore that 's important done study For examine how the mathematization process works student in finish problem algebra . Through analysis answer written , think aloud, and interview , researchers can identify steps taken students , the difficulties they experienced , and the strategies used . The results of study This expected can give description comprehensive about pattern think students , at the same time become input in designing learning more algebra contextual , meaningful , and supportive development literacy mathematics student .

## 2. Methodology

Study This use approach qualitative with design studies case exploratory research This aim For describe the process of mathematization progressive student in finish question algebra . Subject study obtained with method give related questions algebra to 26 students of SMPN 2 Balusu Toraja Utara for the 2024/2025 Academic Year . Furthermore, there are 3 students were selected who will the mathematization process is described . Subject study consists of from 3 students chosen purposively selected based on pattern the answer.

In research qualitative , researcher is instrument main . This means , the researcher himself who directs , observes, and interprets the process. However , for help , used a number of instrument supporters like :

1. Algebra Problem Solving Test – students are asked to work on algebra problems designed to elicit thinking processes from the concrete to the abstract.

2. Think Aloud – while working on a problem, students are asked to describe what they're thinking. This method helps researchers capture strategies and thought processes that aren't readily apparent from the final answer.
3. Interviews – after the test, students were interviewed to clarify the reasons why they chose a particular step, as well as to explore any difficulties they experienced.

Next, the data is analyzed with simple steps:

1. Summarize data from test results, think aloud recordings, interviews.
2. Presenting data in the form of transcripts or tables containing the steps of student thinking.
3. Draw conclusions by looking at how students go through the stages of progressive mathematization.

To ensure reliable results, researchers used triangulation techniques, comparing data from tests, think-alouds, and interviews. They also conducted member checks, confirming the findings with students to ensure they aligned with their experiences.

## 3. Results and Discussion

### *3.1. Results*

In this study, the questions given to students involved the application of algebraic concepts to solve problems related to the perimeter of a rectangle. The research subjects consisted of three students representing high-ability (S1), medium-ability (S2), and low-ability (S3). The following illustrates the mathematization process by S1, S2, and S3.

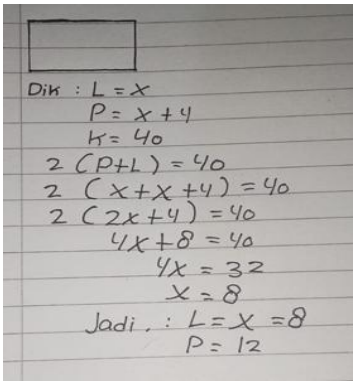


**Figure 1:** Subject S1's Answers

Based on the results of the S1 work, it can be seen that students are able to understand the problem, formulate the algebraic model correctly, solve the equations coherently, and reflect on the answers. This pattern shows that students have gone through the horizontal and vertical mathematization process completely. The results of the S1 think aloud: the question says the length is 4 cm more than the width, meaning the width is x and the length is x + 4. The circumference is 40, meaning 2 (p + l) = 40. I entered the values to get a width of 8 and a length of 12, and to make sure I checked the results.

The following are the results of the researcher's interview with the S1 subject.

Q: Why did you choose width = x?

S1: Because the width is smaller, so I used x for the width, and the length is automatically x + 4. Q: How do you make sure your answer is correct? S1: I checked again with the circumference formula, and it turns out the result is exactly 40.

As for the S1 think aloud data:

"The problem says the length is 4 cm more than the width, meaning the width is x and the length is x+4. The circumference is 40 which means 2(p+l) = 40. I entered the value and it got a width of 8 and a length of 12. To be sure, I checked the circumference result again, it was correct 40."

Based on the results of the test, interview, and think-aloud, S1 showed a complete horizontal and vertical mathematization process , namely being able to understand the problem well, choose the right strategy, complete the calculations sequentially, and recheck the answers to ensure the results are correct.

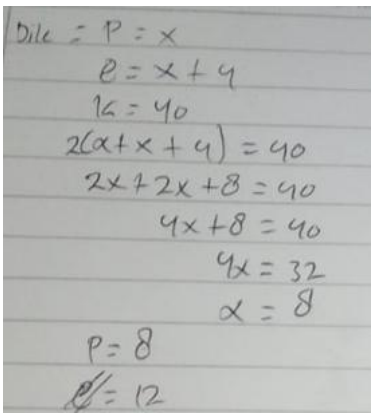


**Figure 2.** Subject S2's Answers

S2 answered, it appears that the student partially understood the problem but made a mistake in determining the variables. The algebraic solution was still performed, but the final result was inconsistent with the context of the problem.

The following are the results of the researcher's interview with S2

Q: Why did you use x for length, not width?

S2: I thought because the length was mentioned first, so I used xP: After getting the length 8 and the width 12, do you think it matches the question? S2: Just realized, the length should be greater than the width. So it seems like it's the other way around

The data on think aloud of S2 students:

"I took the length = x, because the length was mentioned first in the problem. So the width is x – 4. The circumference is 40, which means 2(x + (x – 4)) = 40. From there, I calculated that the length is 12 and the width is 8. Eh… but if I look again, the length is actually smaller than the width. So it seems like it's the other way around."

Based on the results of the test, interview, and think-aloud, it can be concluded that S2 is capable of horizontal mathematization (converting problems into algebraic models), but incomplete in vertical mathematization due to errors in variable assignment. Conceptual errors occur in understanding the relationship between variables, but students are still able to reflect and recognize the discrepancy.

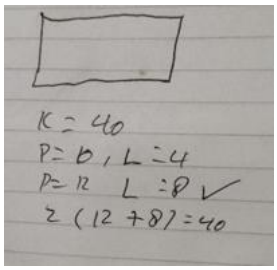


**Figure 3.** Subject S3's Answers

From S3 answer, it is clear that the student did not create an algebraic model, but instead directly guessed the number and tested it with the circumference formula until he found the right answer. The following is the result of the researcher's interview with S3.

Q: Why did you immediately determine the length and width?

S3: Because I don't know the formula, so I just guessed it. Q: How do you ensure the answer is correct? Student: I calculated the circumference again, and it turned out to be correct.

As for the think aloud data for subject S3:

"I was confused about the formula, so I just experimented. If the width was 10, the length was 14, and the circumference was 48, that was wrong. I tried again, the width was 8, the length was 12, and the circumference was 40, and that was it. So that was the answer."

Based on the results of the tests, interviews, and think-alouds, it can be concluded that 3 did not go through an algebraic mathematization process, but rather through trial and error. His thinking process was intuitive and based on guesswork, rather than symbolic representation. Although the final result was correct, his thinking demonstrated limitations in using algebraic concepts.

**Table 1.** Overview of the Mathematization Process for S1, S2, S3 Subjects

| Subject | Mathematization Category | | | |
|---|---|---|---|---|
| | **Understanding the Problem** | **Formulating a Model** | **Solution to problem** | **Reflection** |
| S1 | Student understand formula circumference and relationship length and width | Formulate a model with Correct that is : Width = x, length = x + 4 | Able to solve problem with correct procedure4x + 8 = 40 x = 8 length = 12 | Check again : 2(12 + 8) = 40 |
| S2 | Understand formula around , but Confused variables length and width | Wrong in do modeling | Do calculation with correct 4x + 8 = 40 x = 8 length = 8, width = 12 | Not aware mismatch with condition question |
| S3 | Difficulty understand problem | Not creating an algebraic model | Trying it out numbers : length 10, width 6 so circumference 32 (wrong); length 12, width 8 so circumference 40 ( correct ) | Answer obtained from try it out , without formal reflection |

### *3.2. Discussion*

Findings study This show existence variation pattern think student in do mathematization progressive when finish question algebra . Variation This show that the thinking process student influenced by the level understanding concept , experience study , as well as their strategies use in face problem contextual . Pattern A describes students who have is at the formal stage . They capable connect context problem to in mathematical models with good , arrange equations , solve in a way sequentially , and do reflection For ensure truth answer . This pattern in line with draft *mathematization progressive* proposed Treffers (1987), namely transition from horizontal mathematization ( connecting problem contextual with representation mathematics ) towards mathematization vertical ( completion use rule formal symbolic ). Research Gravemeijer & Doorman (1999) also emphasized that ability For move from context to formal representation is characteristics main success learning based *Realistic Mathematics Education (RME)* . Findings similar shown by research latest *Algebraic Thinking Process of Students with High Mathematical Ability* (2024) who found that student with ability tall more easy do transition from informal to formal representation in settlement linear equations .

Pattern B shows existence understanding partial . Students Already try build mathematical models , but Still there is misconceptions , especially in determine variable . Error kind of This common found in learning algebra Because student tend fixated on structure Language question than understand relation mathematics contained in it ( Kieran, 2007). Although Thus , the effort student For forming models and operating algebra Already visible , only just reflection to results Still less than optimal. Findings This in line with Stacey & MacGregor's (1999) research found that that difficulty main student in algebra is interpret symbol , not solely do manipulation procedural . This result is also supported by research the latest in Indonesia, for example *Students' Misconceptions in Simplifying Algebraic Expressions Based on Assimilation and Accommodation Frameworks* (2023/2024), which shows that student often do error in determine variables , substitution , and *reversal error* . Likewise , *Students' Misconceptions in Algebraic Concepts: A Four-Tier Diagnostic Test* (2022) found that Lots student Still trapped in misconception although in a way procedural they can finish question .

Pattern C reflects more students Lots depend on intuition and trial and error strategies . Although these strategies sometimes produce answer right , the process is carried out No through track formal mathematization . This signify existence gap between understanding conceptual and understanding procedural (Hiebert & Lefevre, 1986). However , the pattern This still important noted because informal strategies can become point beginning going to modeling more algebra systematic . Cobb et al. (1992) emphasized that informal strategies do not may ignored , but rather made into foothold For guide student to direction formal representation . In line with that , research latest *Exploring Learner Errors and Misconceptions in Algebraic Expressions with Algebra Tiles* (2024/2025) proves that use of manipulative media can help student switch from trial and error strategy going to understanding formal. This is indicates that approach learning based experience concrete Can become bridge for student in transition mathematization .

In a way general , results study This confirm that mathematization process progressive No walk with pattern single . There are students who immediately can linking context with formal models, some still do experience obstacles at the stage beginning . This is show importance the role of teachers to give room exploration of diverse strategies , while guide students to be able to reflect results Work them and improve the models that are still wrong . With Thus , learning algebra No only emphasize on procedures resolution , but also strengthening literacy mathematical through understanding the meaning behind symbol (OECD, 2019). Research latest in Indonesia ( *Analysis Mathematical Literacy Skills Student*

*through Solution Problem Algebraic Expressions* , 2022) also shows that although student Enough Good in formulate problem to the mathematical model , they Still weak in interpret results and relate them with context .

Findings about variation pattern think student give description real for teachers in designing learning algebra . The teacher does not can assume that all student will follow track think the same . Instead , they need realize that There is students who have Can direct connect context to a formal model ( as in Pattern A), some still stuck at the stage half mature (Pattern B), and some are moving with intuitive and trial and error strategies (Pattern C).

From here , teachers can take steps following : (1) Giving room strategic exploration : thinking process with various method still need appreciated . Let student try with their respective strategies, then point For compare strategies so that they Study from diversity approach (Cai, Jakabcsin , & Lane, 1996), (2) Guiding transition from informal to formal: for students who still make mistakes in determining variables or Confused modeling problem , teachers can exemplify method change situation daily become equality simple . This process help they move from horizontal stage to vertical ( Treffers , 1987), (3) Pushing reflection to answer : many student stop after find results , even though Not yet Of course in accordance context . Teachers can practice student For return to questions and check whether results obtained logical (Polya, 1945), (4) Linking with literacy mathematics : learning algebra No just exercise symbols and formulas , but also necessary involving close context with life students . With method this , students more sensitive to meaning from the mathematical models they build (OECD, 2019; Cahyani, Shodiq , & Agustin, 2022). With notice variation pattern think students , teachers can designing more learning strategies adaptive , so that the mathematization process progressive truly support development reasoning mathematical student .

Furthermore, the variety of thinking patterns found in this study reaffirms the importance of the zone of proximal development concept (Vygotsky, 1978) in the context of algebra learning. Students in Patterns B and C actually have the potential to develop formal thinking, but they require appropriate pedagogical intervention. By facilitating collaborative interactions, group discussions, and teacher scaffolding, the transition from intuitive strategies to formal mathematization can be smoother. This demonstrates that the development of mathematical reasoning is not a static process, but rather a dynamic process and is heavily influenced by social interactions and a conducive learning environment (Rahmawati & Yuwono, 2022; Tanujaya, 2023; Widjaja & Heck, 2024).

The findings of this study enrich the discourse on the importance of multiple representations in mathematics learning. Kaput (1991) emphasized that students' conceptual understanding will be more solid if they are able to shift between representations whether verbal, visual, symbolic, or numerical. Pattern A demonstrates success in connecting contextual representations to formal symbols, while Patterns B and C indicate the need to strengthen the bridge between these representations. Therefore, a learning strategy based on representational fluency is a potential solution to address gaps in students' understanding of progressive mathematization (Cahyono, Nusantara, & Sa'dijah, 2022; Arsaythamby & Shahrill, 2023; Simamora, 2024).

The variation in thinking patterns also opens up space for further research into students' cognitive profiles in learning algebra. By using a more in-depth qualitative approach, such as think-aloud protocols or eye-tracking analysis, researchers can uncover more detailed internal processes related to how students interpret contextual problems, construct mathematical models, and verify the correctness of their answers. Such an approach is not only useful for understanding individual student differences but also provides an empirical basis for developing more adaptive learning trajectories. Thus, this study not only identifies variations in thinking patterns, but also opens a new direction for pedagogical studies in order to strengthen the process of progressive mathematization in algebra classes (Sari, Wijaya, & Retnawati, 2023; Abdullah & Harun, 2024; Liu, Li, & Zhang, 2025). 4. Conclusion

The meta-analysis of 20 studies shows that Dynamic Mathematics Software (DMS) has a large and consistent effect (ES ≈ 0.69) on enhancing students' creative thinking in mathematics. The impact remains stable across years, educational levels, and types of digital technology, with no significant differences found. This indicates that the success of technology-assisted learning depends more on pedagogical strategies and teacher competence than on the specific software used.

## 4. Conclusion

The results of this study clearly show that each student has a different approach to solving algebra problems based on rectangles. Some students are able to work sequentially, starting from understanding the problem, creating a model, and then checking the results. Others actually understand the core of the problem but still make mistakes when creating the model, resulting in inconsistent answers. Meanwhile, others choose shortcuts by guessing numbers until they find the right result. These differences in patterns indicate that the mathematical thinking process does not proceed in a straight line or the same way for everyone. Instead, this variation illustrates that progressive mathematization skills develop gradually, from mere trial and error to finally being able to construct a correct model.

Based on these findings, there are several recommended suggestions, namely: (1) It's important for teachers to give students the opportunity to try different approaches, but they should still be guided so they learn to model problems appropriately. Classroom discussions can be helpful, as students can see differences in strategies and learn from each other, (2) Students need to get used to not only finding the right answer but also understanding *why* that method was used. Creating a model with variables will help them be more systematic than simply guessing, (3) For future researchers, this

research can be continued with more varied questions, for example on quadratic equations or systems of equations, so that the picture of students' thinking patterns becomes richer.

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